Why Mathematics is Large

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1 Program

The following are at least candidates for generically wild maps, over enough of STEM Theory to render STEM Theory large.

1) *Arenize* ⟨2-8⟩

$$o \longrightarrow \mathfrak{O} .$$

This sends any type of Mathematical object to the Mathematical space formed by the totality of those objects. Generalizing the notion of configuration space in Physics and Dynamics to any kind of Mathematical object. Alias forming the moduli space. There are plenty of instances of this mapping between traditional branches of mathematics. I.e. of $\mathfrak{O}$ not belonging to the same traditional branches of Mathematics as the corresponding o .

'Summing, integrating, placing measures over, averaging or tracing' over this places a Measure-Theoretic arrow on this bow.

2) *Quotientize* ⟨2-7⟩

$$o \longrightarrow o/\!\sim .$$

Because this generically does not preserve the type of object o . This compounds with 1) in the form of quotient arenas

$$\mathfrak{O} \longrightarrow \mathfrak{O}/\!\sim .$$

Quotient too little and insights are lost. Quotient too much and the quotient arena becomes Mathematically intractable.

Examples of quotients include orbit spaces. And manifolds quotienting not to manifolds but to Whitney's stratified manifolds.

3) *Dualize* ⟨2-7⟩. A prototype here is the Fourier transform. Pontryagin duality, spectral inversion and the Zariski duality of commutative rings are more sophisticated examples. A wildness in the last of these is endemic non-Hausdorffness.

4) *Automorphosize* ⟨3-8⟩

$$o \longrightarrow Aut(o) .$$

Which is not to be confused with symmetrizing an object. One usually first encounters this by passing from groups to their automorphism groups. With extension to other simple Algebra objects. The top end of our stated range covers for instance finite Projective planes' and Riemann surfaces' automorphisms.

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⟨X⟩ is the level of a topic: approximately which university year the topic is covered.

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5) *Completize* ⟨3-8⟩ A prototype here is the graphon continua corresponding to graphs. Renormalization group flow is another, now originating in Theoretical Physics. Some indications of wildness here are the *Cantor set*: fractal. The *blancmange function*: everywhere-continuous and yet nowhere-differentiable. And *space-filling curves*.

6) *Desingularize* ⟨3-9⟩ These are alias blow-ups, resolutions and unfoldings. Usually first encountered for covering spaces. Singularities in a broad Geometrical context turn out to be Lie-algebra-valued and thus Dynkin-diagram-classifiable. Sanitizing some pathological quotients is part of this, for all that not all circumstances motivate such sanitization.

7) *Quantize / Deformize* ⟨2-8⟩

$$o \longrightarrow \hat{o}$$

for instance with half of the variables becoming operators which bring in a fixed constant. Which in QM is usually denoted $\hbar$ (or more strictly a non-dimensionalization thereof). And then features as a multiplicative factor in nonzero commutators.

8) *Internalize* ⟨5-6⟩

$$o \longrightarrow \underline{\mathfrak{Cat}}(\mathfrak{Rel}(o)) \ .$$

With reference to the category formed by the internal relations of o . This is not to be confused with assigning categories to the objects themselves.

An extra piece of 5) at the Categorical level is a corresponding notion of the forming limits.

9) *Localize* ⟨5-9⟩ alias inversion in a certain sense. From a ring to a prime ideal is an Abstract Algebra prototype.

10) *Universalize* ⟨5⟩ in the sense of universal enveloping algebras, alias the free set-up. From an algebra's generators to universal enveloping algebras is now an Abstract Algebra prototype.

11) *Homotopize* ⟨7-9⟩

$$o \longrightarrow \pi_n(o) \ .$$

Algebraic Topology and Homological Algebra are settings for this. It brings about a unification of Algebra with Topology and Logic. Even the p-spheres $\mathbb{S}^p$ exhibit sequences of higher homotopy groups that are not known to possess a closed form.

Homological algebra provides a further setting for 6). While inverting weak equivalences to form homotopy categories is a further case of 9)

12) *Tropicalize* ⟨9⟩

$$o \longrightarrow \mathfrak{Trop}(o)$$

is alias zooming out, coarsening or dequantizing.

This sends Differential Geometry to Combinatorics in a distinctly novel way. Geometrical Group Theory exhibits a parallel. This is wild along the lines of wildly-ramified valuations in Number Theory. And wild undecideability and chaos in Abstract Algebra.

Many of our names for the above are new. Though often only at the level that we have jointly stylized them to end in 'zed'... For mutual recognition, modulo a list that we shall shortly begin to curate of mathematical 'izes' that are not of this ilk. And then for mutually-signposted notation: our dozen 'izes' are all to have matching italic-font map names ending in 'ize'.

I.e. a first twelve reasons that Mathematics is large are:

$$Arenize, Quotientize, Dualize, Automorphosize, Completize, Desingularize,$$

$$Quantize \text{ or } Deformize,$$

$$Internalize, Localize, Universalize,$$

$$Homotopize \text{ and } Tropicalize\ .$$

The first 7 can be initiated pre-Categorically, for all that Category Theory strengthens them in many ways. The next 3 are Categorical. The final two involve extra Topological content, and a Geometry to Combinatorics bridge respectively.

A reason for presenting this now is as follows. That the Ages of the world in which small prescribed lists of longstanding conjectures are now over. Due to large amounts of claimed proofs by big AI companies in 2026. One remedy for this is making conjectures about the new landscape created by what has been proven or disproven. But another is a reminder that Mathematics was always in any case large. So there always has been a vast amount of room for finding one's own examples and making one's own conjectures.

Acknowledgments I thank Jan Saxl, Chris Isham and S. Sánchez for discussions,long past, and Gemini for some checks and scouting. This Encyclopaedia v0 entry is a stub. Expect a skeleton crew of references in the next update!

A v0.1's context for what v0 presented as the above program

What we have done here so far is to establish a bridgehead list. With a bit more time, we can systematically address conceptually what each map is. Where it occurs up to a fixed level in current undergraduate and then graduate courses. That precisely what 'generic' and 'wild' may mean differs from map to map across this list. While giving basic examples of types of wildness encompassed, as occur sporadically in current undergraduate and then graduate courses.

The situation being described here is that *isolated* examples of these maps are common in current graduate courses. And not unheard of in some cases in undergraduate courses. But these are usually presented as tricks, without asking how they generalize over any of the other areas of the theory of STEM. While most of them apply in between many of the currently explored areas of STEM, and *all* areas of STEM, known or unknown...

♠ ◇ ♣

For my own background on this matter, I am an expert in *Arenize* . I have described *Arenize* , and have detailed *Arenize* in various STEM fields. Including novel work on *Arenize* starting from the most very basic topics in STEM Theory. As such, I will not single out *Arenize* for specific examples again here, pointing soon instead to references for this topic. Rather I am starting to review how *Arenize* is not the only source of very plausibly generically wild mathematics. Whose wildness is generative of new Mathematics from familiar Mathematics.

Instead, I point out that I took *Arenize* far enough to encounter its overlap with some of the other maps. In particular, *Quotientize* in the context of quotient arenas. And then *Desingularize* on some of the resulting irregularity and wildness.

Two of my past mentors introduced me to the universality of one map on this list each. I thank Jan Saxl for getting me to take *Automorphosize* everything seriously. While many of the reasons that I initially took *Arenize* everything seriously come from its role as a prerequisite for the following. Isham's taking *Quantize* everything seriously. Alongside, in my background reading, Kendall's *Probabilize* everything seriously (which is a candidate for a 13th map). While Classical Physics and Linear Algebra provide separate bastions for taking *Arenize* seriously prior to any of these.

My awareness that *Dualize* , *Completize* , *Internalize* , *Localize* and *Universalize* usefully extend to everything has mixed origins. S. Sánchez taught me to systematically track conceptual types. Isham motivated me to learn Category Theory proper, and so I attended Johnstone's lectures and read several books on this. S. Sánchez pointed me to *Internalize* paralleling some features of *Automorphosize* . And got me to take *Localize* and *Universalize* seriously.

Isham, and my taking Ghrist's book as Discussion Group topic to S. Sánchez motivated me to roughly quadruple the amount of Topology in my notes. This was the backbone of my becoming an expert in *Arenize* , since asking what Topology each arena has is always a good question. These were the circumstances under which I learned to take *Homotopize* everything seriously.

Disambiguating *Dualize* and *Completize* from unrelated duality and completion uses required my use of Gemini in 2026. So as to scour the literature, the concepts and the naming. This scouting run also satisfied me that *Tropicalize* belongs on this list.