The Pappus Incidence Graph

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Abstract

Pappus' Theorem is one of the two main structural theorems of Projective Geometry. Its configuration can be modelled as a graph. Projective Geometry is furthermore Geometry pared down to the study of incidence, and incidence graphs are themselves meaningful. This brings the Pappus incidence graph to the forefront. We set it up as a bipartite graph and study its basic Graph Theory properties. We also use it to give further examples of Graph Theory drawing and visualization, including use of shellings, grids and other tessellations.

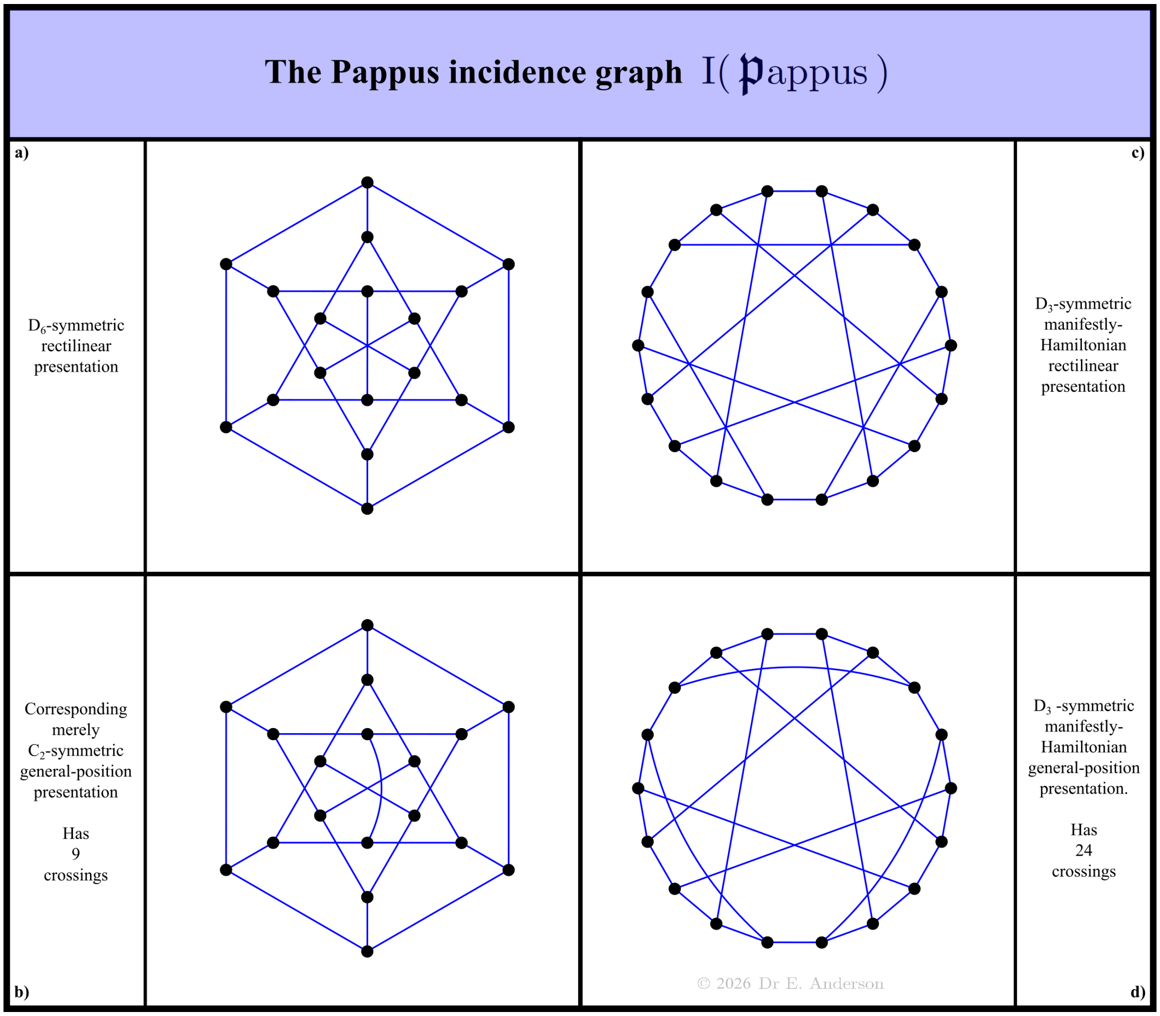

Figure 1:

This Article is ⟨3⟩: accessible to third-year undergraduates.

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1 Introducing Pappus' Incidence graph

1.1 Motivation

Motivation 1 This is a further Article on abstracting interesting graphs from *Projective Geometry* [20, 10, 11, 3, 8, 12] : the study of Geometry pared down to incidence. We previously considered [34] Pappus' configuration and theorem [1, 9, 11, 8, 27, 22]. And the *Pappus configuration graph*: directly modelling the 9-point configuration itself.

Remark 1 We now consider instead the corresponding incidence graph.

Definition 1 Suppose that we are given an Incidence Geometry configuration. Then the corresponding *incidence graph* alias *Levi graph* has as its vertices (blue) each primary vertex (grey) and line in the configuration. Pairs of these blue vertices are furthermore accorded edges whenever the corresponding primary vertices or lines are incident.

Motivation 2 Since Projective Geometry is the study of incidence, it makes particular sense both to consider incidence graphs for Projective configurations and to use this name in this context.

Remark 2 While we call the current Article's object of study [6, 21, 18, 23] the *Pappus incidence graph* $\mathrm{I}(\mathfrak{P}\mathrm{appus})$; be warned that many other sources [35, 36] refer to this one as the Pappus graph...

1.2 Bipartite inception of our incidence graph

Structure 1 We give this in Fig 2 using the previous Article [34]'s call-signs for the Pappus configuration's objects. Firstly using uniform horizontal spacings in Subfig a). Which however exhibits crossings not in general position: a common Topological consideration. We then resolve this in Subfig b) on a natural-numbers grid, just not with every slot occupied by a vertex! See Appendix A.1 for the intermediary yoga.

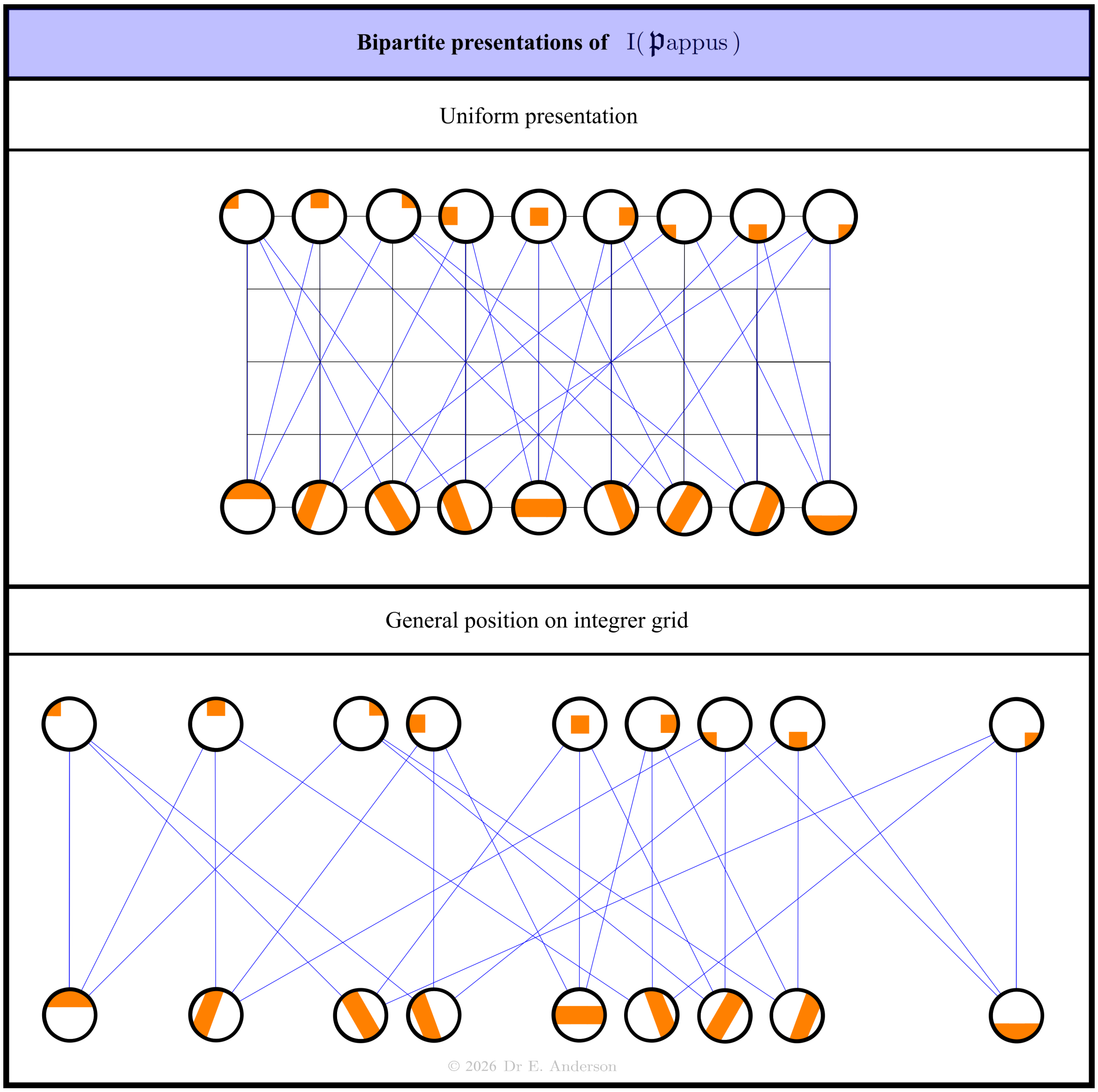


Figure 2:

1.3 A D_6-symmetric presentation

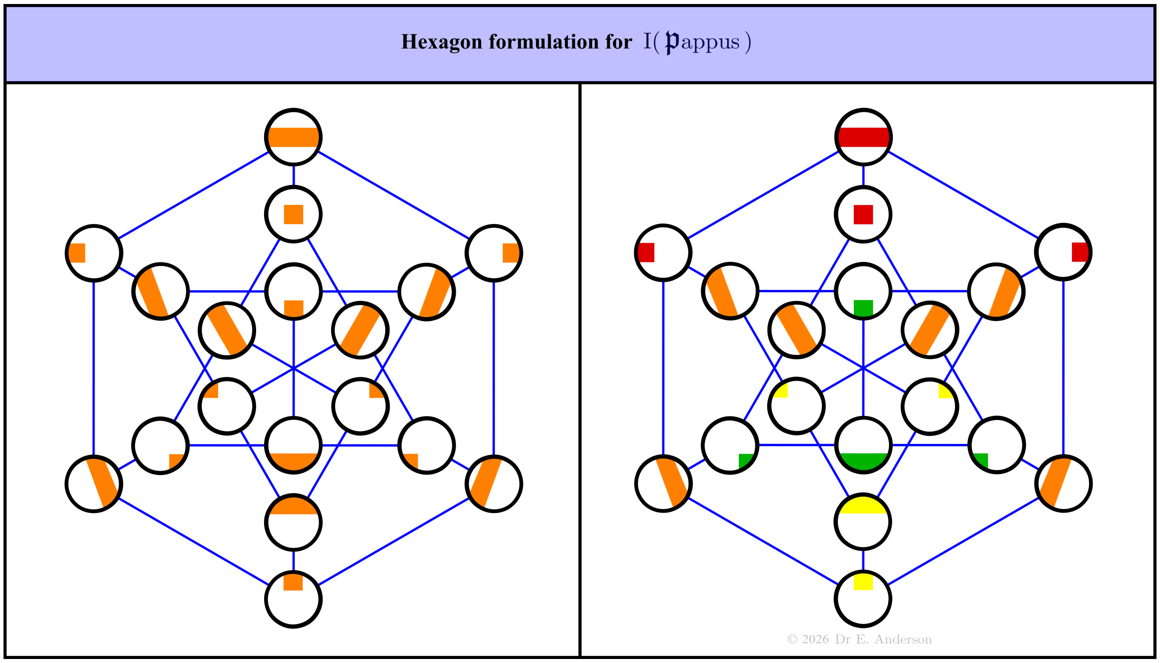


Figure 3:

Structure 2 Let us next pass to a widely used presentation of this famous graph: Subfig 1.a). The call-signs themselves establish the isomorphism that we use in passing between these graph presentations: Fig 3.a) This presentation manages to exhibit somewhat more symmetry than the others in our cover-figure. This presentation features for instance in [36] and, with 18 edges curved into arcs, in [35].

Remark 3 This said, the Pappus configuration and incidence graphs have rather more symmetry than this presentation manages to muster. With automorphism group

of order 108 , or, including dualities as well, of order 216 .

Pointer 1 See [13, 14] ⟨6-8⟩ if interested in which specific group this is. In contrast, the current presentation's dihedral group D_6 is just of order 12 .

Remark 2 Subfig b) deals with this presentation having a common Topological problem, namely that its crosings are not in general position. This is manifested by 3 edges meeting at a non-vertex point, which we place at the centre of symmetry. We deal with this by bending one edge. This is a presentational technique that is already familiar to most Readers from the presentations of the also famous utilities graph

$$\text{Utilities} \;=\; K_{3,\,3} :$$

the complete bipartite graph with equal parts of size 3 . This comes at the cost of breaking most of Subfig a)'s presentation's manifest symmetry.

Remark 3 Among the many possible D_6-symmetric presentations, the displayed configuration has the following characteristics. It consists of 3 concentric hexagonal shells. Equally radially spaced both mutually and relative to the centre of symmetry. This is an example of a 'shelling', 'layering' or 'onion' property of a graph presentation (Appendix A.2).

Remark 6 Subfig 3.b) colour-codes the Pappus input objects in yellow and green, and the outputs in red. These each form a claw subgraph. Subfig a)'s presentational symmetry is maintained by this colouring.

1.4 Some basic counts

Remark 1 Having given one presentation so as to be able to display our graph, let us make the following basic counts for future use. Its number of vertices is

$$V(\mathrm{I}(\mathfrak{P}\mathrm{appus})) = 18 . \tag{1}$$

Its number of edges is

$$E(\mathrm{I}(\mathfrak{P}\mathrm{appus})) = 27 . \tag{2}$$

And its degree sequence is

$$\deg(\mathrm{I}(\mathfrak{P}\mathrm{appus})) = 3^{18} . \tag{3}$$

So all of its vertices' degrees are the same, which Graph Theorists call *regular* [29]. And their common value is here 3, which Graph Theorists call *cubic*, and cherish [4, 5, 7, 15, 30]. This 3 carries Projective significance; in further detail, the underlying Projective configuration is 9_3. Signifying 9 points arranged to enjoy collinearity in 3's. At the level of the incidence graph, then, this enforces the cubic subcase of regularity...

Remark 2 There is no practical point to considering the complement of our graph, since this has far more edges than our graph itself. This occurs whenever

$$E << E_{\max} = \frac{V(V-1)}{2} .$$

For us,

$$E_{\max} = \frac{18 \times 17}{2} = 153 >> 24 = E . \tag{4}$$

So

$$E\left(\overline{\mathrm{I}(\mathfrak{P}\mathrm{appus})}\right) = 153 - 27 = 126 .$$

Exercise 1$^-$ Check this number using regularity instead.

Remark 3 A finer consideration, for drawing and naming purposes, is to entertain whichever of G or $\overline{\mathrm{G}}$ is of smaller size ($=$ edge number). Which is to be gauged against the critical value

$$E_{\mathrm{crit}} := \frac{E_{\max}}{2} = \frac{V(V-1)}{4} . \tag{5}$$

For us, this is

$$E_{\mathrm{crit}}(\mathrm{I}(\mathfrak{P}\mathrm{appus})) = \frac{18 \times 17}{4} = \frac{153}{2} = 76.5 . \tag{6}$$

Then indeed

$$27 < 76.5 < 126 . \tag{7}$$

So we go with drawing, and naming, the 27-edge version!

Remark 4 Projectively significant graphs span both sides of the divide as regards whether complements are worthwhile. For the Fano graph [33], they are. For the Pappus graph [34], the count is the critical value itself. For all of the Desargues graph [38], and the 3 corresponding incidence graphs ([37, 39] and the current Article; see also [6, 23]), however, using the complements is less advantageous. The counting-level technicality here is that these graphs are both too large and too sparse to have useful complements.

2 Further properties

2.1 Metric properties

Definition 1 The *girth* $g(\mathrm{G})$ of a graph G is the length of its smallest cycle. While the *circumference* $c(\mathrm{G})$ of a graph G is the length of its longest cycle.

Definition 2 The *diameter* $diam(\mathrm{G})$ of a graph G is given by the following. The maximum over all vertex pairs of the minimum lengths between each vertex pair.

Remark 1 Throughout the above definitions, length refers to path-length as measured by the number of edges in that path. Trees are accorded infinite girth by convention.

Exercise 2 Show that

$$g(\,\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)\,) \;=\; 6\;, \tag{8}$$

$$d(\,\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)\,) \;=\; 4\;. \tag{9}$$

Why are we not explicitly noting down $c(\,\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)\,)$ in the current Article?

2.2 Structural analysis

Remark 1 No cubic graph contains any degree-1 or -2 vertices. Thus all cubic graphs are both foliation irreducible and homeomorph irreducible. Which combination we term double irreducible: class D [32].

Remark 2 $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is a 7th homeomorph of the first cubeomorph irreducible: the tetrahedron graph $\mathrm{Tet} = \mathrm{K}_4$.

Exercise 3 [28] a) Show that $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is a 6-cubeomorph irreducible but not a 5-cubeomorph irreducible. Where a *g-cubeomorph* means a girth-g preserving cubeomorph.

b) Show that $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is a cubeomorph of the Dürer graph and of the Petersen graph.

c)$^+$ Show that $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is not however a cubeomorph of the Heawood graph.

2.3 Traversibility properties

Remark 1 $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is clearly not Eulerian. For by (3) it has all vertices of odd degree. But Eulerian graphs must have all vertices of even degree!

Remark 2 Consider again the symmetric presentation of Fig 1.a). In Fig 4, we mark upon this in emerald one of the Hamiltonian cycles that is conceptually simplest to describe. I.e. view the vertices as lying on 3 hexagons. The inner 2 of which have overlapping edges and are depicted as triangles with marked midpoints. First go around the outer hexagon as far as possible. Next cut in to an inner hexagon and sweep out all of its vertices. Then cut across into the other inner hexagon and sweep out all of its vertices. This finally has the good fortune of leaving us exiting back to the outside in the right place to close up the overall cycle.

Remark 3 Subfig 1.c) instead manifestly exhibits that $\mathrm{I}(\,\mathfrak{P}\mathrm{appus}\,)$ is Hamiltonian. We use callsigns to establish the isomorphism between these presentations, in Fig 5.a). On this occasion, one has the good fortune that most of Subfig a)'s manifest symmetry can continue to be exhibited. This presentation features for instance in [36] and, without centering about a symmetry axis, in [35]. It features, centred and yet upside-down relative to our presentation, in the world's first review of Projectively significant graphs: Coxeter's [6].

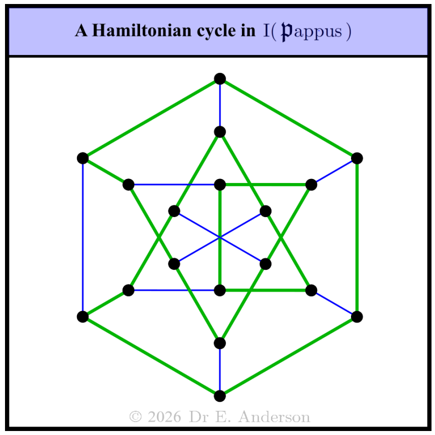


Figure 4:

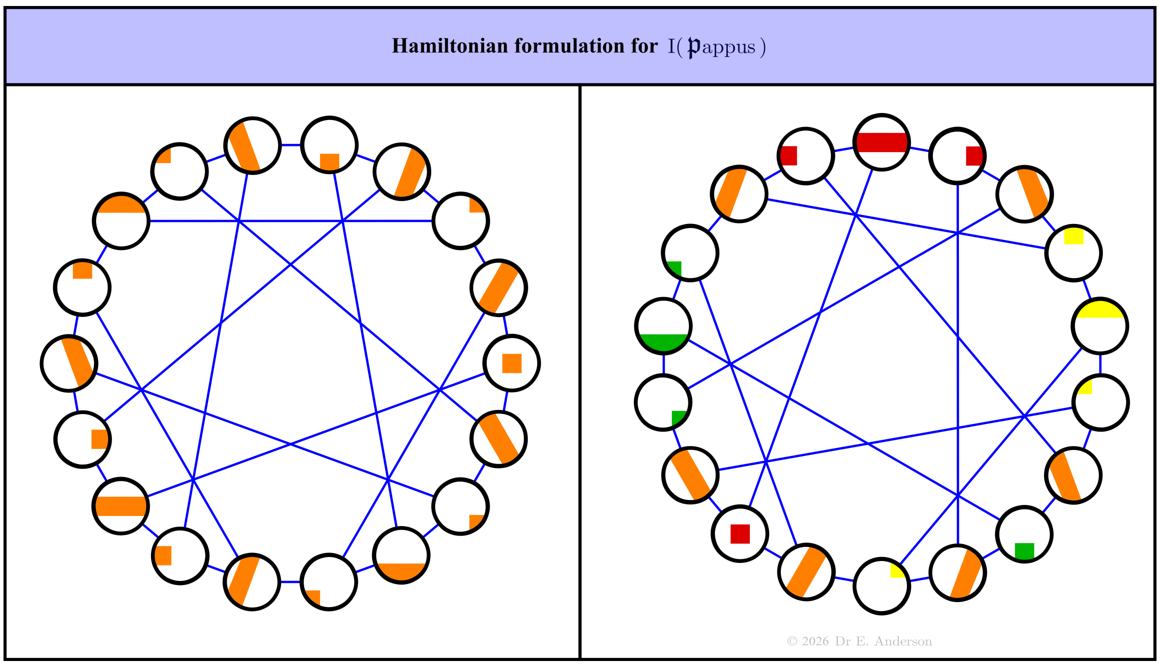


Figure 5:

Remark 4 Subfig c) however suffers from a rather more extensive case of not being in general position. Now no less than 6 separate cases of 3 edges meeting at a non-vertex point are present. Rather neatly, one only needs to bend 3 edges to handle this, nor is any presentational symmetry is lost in the process! See Subfig d).

Exercise 4$^-$ Retain a larger chunk of symmetry by bending more than one edge in Subfig a) in the process of getting into general position.

Exercise 5 Investigate whether Fig 1.a)'s presentation permits other shapes of Hamiltonian cycle. And equivalent colorations with one input colour and the output colour interchanged. Are these distinct as cycles within presentation-free graphs? In the process, figure out where Subfig c)'s nicely symmetric Hamiltonian presentation can be taken to reside within Subfig a)'s presentation...

3 Minimum-crossing presentations

3.1 Preamble

Exercise 6$^-$ Find a $K_{3,3}$ subgraph within. Why is the other forbidden subgraph for a planar graph, K_5, irrelevant to our analysis?

Definition 1 The *crossing number* Cr is the minimum over all presentations of the number of crossings cr exhibited in each presentation.

Exercise 7 Quickly obtain an 8-crossing presentation from the bipartite graph, and another from the Hamiltonian presentation.

Remark 2 Thus combining Exercises 6 and 7,

$$1 \leq Cr(\mathrm{I}(\mathfrak{P}\text{appus})) \leq 8\,. \tag{10}$$

3.2 Failing to improve the lower bound

Remark 1 In simple cases, we can either draw the graph with even less crossings to tighten the upper bound. Or use an inequality to tighten the lower bound. In the present context, the first is impossible, while an incipient inequality one can always try out for the second is follows.

$$Cr \geq E - E_{\text{planar}} = E - 3V + 6 = E + 3(2 - V)\,. \tag{11}$$

Exercise 8 a)$^-$ Show that this is not useful for $\mathrm{I}(\mathfrak{P}\text{appus})$.

b) Show that a nontrivial amount of girth[1] $g > 3$ improves (11) to the following.

$$Cr \geq E + \frac{g}{g-2}(2 - V)\,. \tag{12}$$

c)$^-$ Show that the good fortune of (8) on this occasion fails to be enough to turn the tide.

d) Show that the following more advanced *crossing number inequalities* [17, 25] ⟨6-7⟩ also fail to help out.

$$\begin{array}{lll} Cr(\mathrm{G}) \geq \frac{E^3}{64\,V^2} & & \text{for} \quad E > 4V\,. \\ Cr(\mathrm{G}) \geq \frac{4}{135}\frac{E^3}{V^2} & - \frac{9}{10}V & . \end{array} \tag{13}$$

In the process, compute and blame $\mathrm{I}(\mathfrak{P}\text{appus})$'s sparseness. Use whichever of the 'per unit vertex' or 'relative to the maximum edge density' quantifications of sparseness that best suit the problem at hand.

3.3 Hexagonal shelling and the equilateral tessellation

Remark 1 Start instead with the 3 concentric hexagonal shell formulation of Fig 1.a). Then relocating a particular innermost-shell vertex to the hitherto unoccupied centre straightforwardly brings the crossing number down to 5 (Fig 6.a).

Remark 2 Thus

$$1 \leq Cr(\mathrm{I}(\mathfrak{P}\text{appus})) \leq 5\,. \tag{14}$$

In fact,

$$Cr(\mathrm{I}(\mathfrak{P}\text{appus})) = 5\,.$$

On the one hand, this is too hard to prove in these early stages of writing up a small Encyclopaedia, On the other hand, the presentation that we have just found is of further interest as a minimum-crossing presentation. So we now put in quite some effort into further optimizing this presentation.

[1] Ignoring the infinite girth convention for trees!

Remark 3 This presentation's vertices fit on the equilateral-triangle tessellation of the plane (Fig 10).

Exercise 9 Maintaining alignment of the tessllation with the vertex hexagons, show the following. That not all of the crossings can however be placed concurrently on an equilateral triangle grid upon which the vertices reside.

Remark 4 Due to this combination of merits and limitations, we award this 'missing link' presentation a silver background.

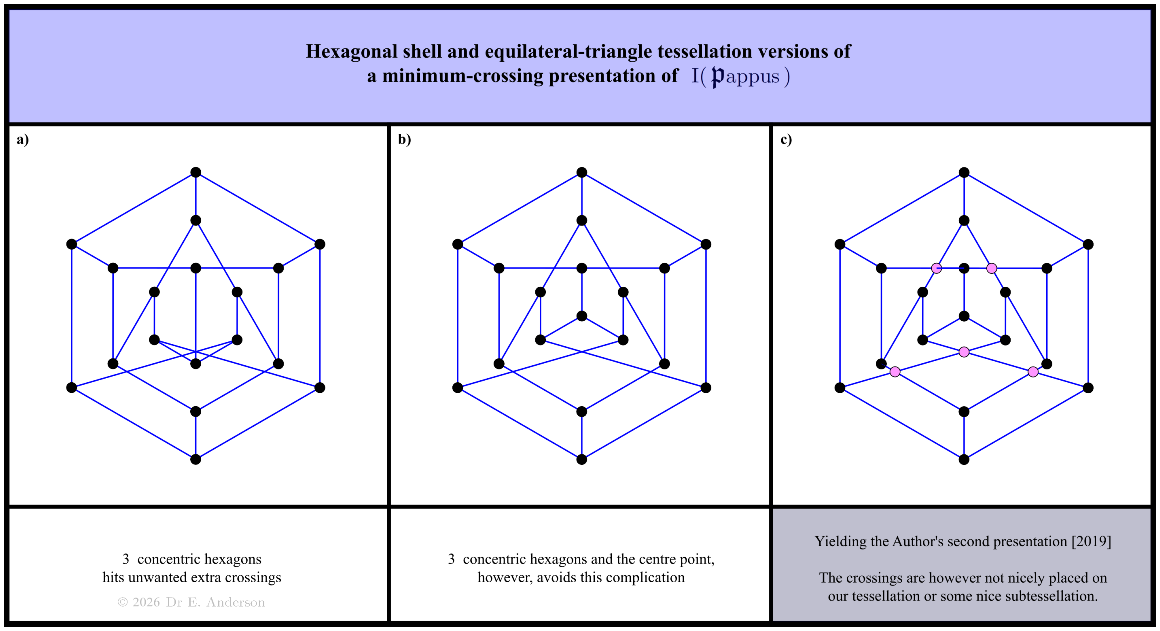


Figure 6:

3.4 Square-grid presentations

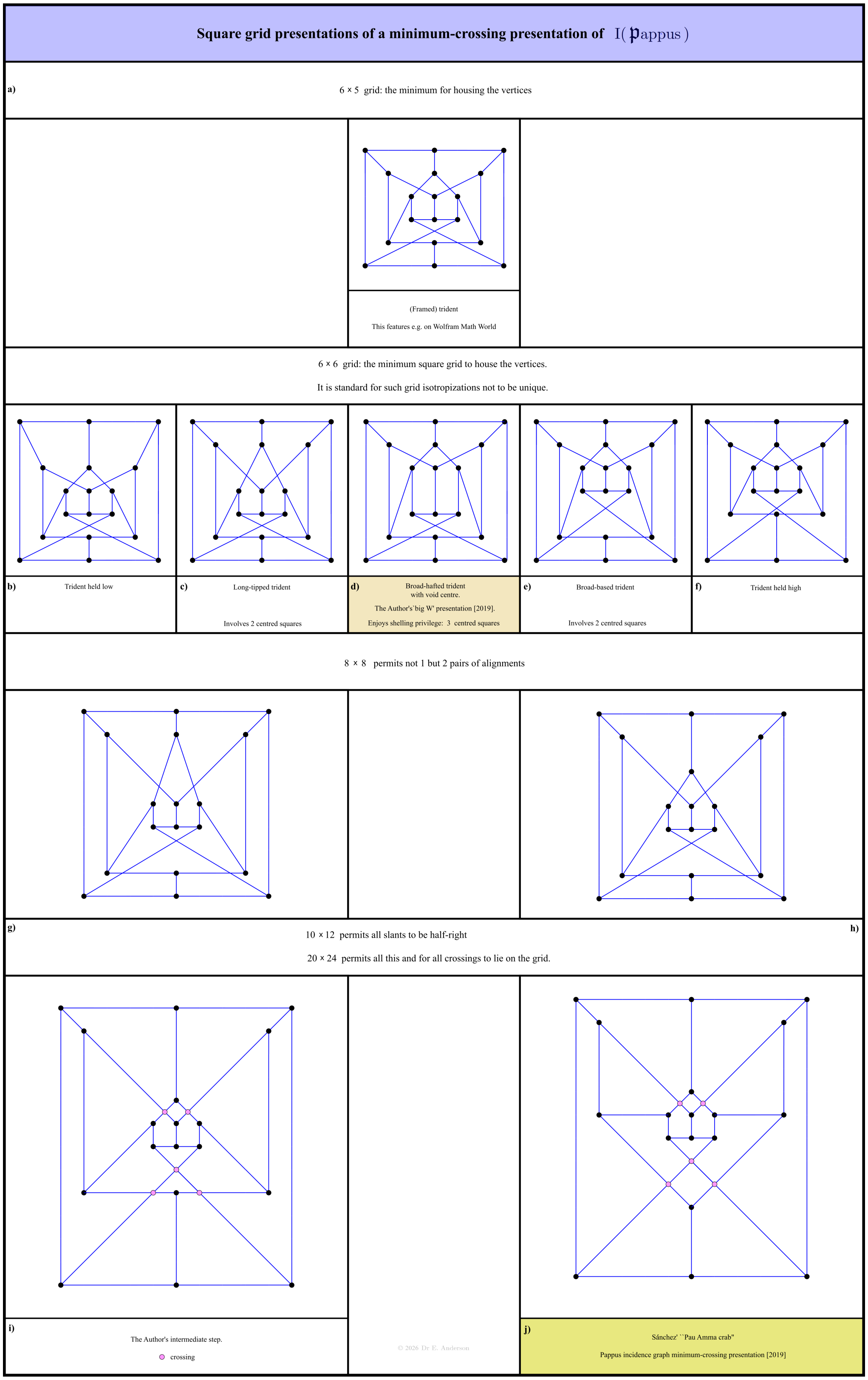
Remark 1 Passing to a rectangular grid, with 2 side-interior points in its outermost shell, turns out to be able to alleviate this. We first bend our crossing-minimum hexagonal presentation into the 'framed trident' of Fig 7.a). Which at the time of writing this up, features on [36] with a clockwise $\frac{\pi}{2}$ rotation. Aside from being a minimum-crossing presentation, this has the virtue of being the minimum square-grid such: 6×5 .[2]

Remark 5 This presentation is not however of square perimeter, which we incorporate in Subfigs b–f): a 5-fold ambiguity. Such redundancies are generic for grid isotropization moves... We award a bronze background to the subcase with the following additional shelling privilege. All grid points here lie on 3 centred squares.

Remark 6 Subfigs g-h) shows that expansion to 8×8 improves the amount of collinearity exhibited.

Remark 7 Subfig j) finally shows that using an even larger grid permits all angles to be right or half-right [28]. Meeting this constraint forces our presentation of I($\mathfrak{P}$appus) to have anisotropic perimeter. A second benefit moreover accumulates: the crossings themselves lie on our grid. Due to

[2] This source does not however lavish this presentation with any of our Remark's attributes.



Square grid presentations of a minimum-crossing presentation of I(𝔭appus)
a)
6 × 5 grid: the minimum for housing the vertices
(Framed) trident
This features e.g. on Wolfram Math World
6 × 6 grid: the minimum square grid to house the vertices.
It is standard for such grid isotropizations not to be unique.
b)
Trident held low
c)
Long-tipped trident
Involves 2 centred squares
d)
Broad-hafted trident
with void centre.
The Author's 'big W' presentation [2019].
Enjoys shelling privilege: 3 centred squares
e)
Broad-based trident
Involves 2 centred squares
f)
Trident held high
8 × 8 permits not 1 but 2 pairs of alignments
g)
h)
10 × 12 permits all slants to be half-right
20 × 24 permits all this and for all crossings to lie on the grid.
i)
The Author's intermediate step.
crossing
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j)
Sánchez' ``Pau Amma crab''
Pappus incidence graph minimum-crossing presentation [2019]

Figure 7:

this, at least out of the selection of 5-crossing presentations currently on offer, this one is awarded 'the gold medal' highlight. S. Sánchez named as on the figure, after its visual and phonetic likeness to the monster crab from the Just So Stories [2]...

Pointer 2 ⟨6⟩ The current section is part of the program [28, 31] of passing from using not only square grids in drawing and visualizing graphs ([26] involves a subcase) but also increasingly general tessellations of the plane [16]. The first port of call here are the tessellations by a single regular polygon. And then by a single less regular tile, or by 2 regular ones.

Acknowledgments I thank S. Sánchez and A. Ford for discussions. And the Applied Combinatorics and Topology Discussion Group members.

A Some presentational finery

A.1 Placing the bipartite presentation in general position

Remark 1 Let us begin with the horizontally-uniform presentation in Fig 8.a).

Remark 2 This fails to have crossings in general position. I.e. some crossing points of edges – other than at vertices – have ≥ 2 edges going through them.

The labelling numbers indicate *excess crossing strength* [28]

$$e := c - c_{\text{generic}} = c - 2$$

for c the actual number of lines going through there. Generically, $e = 1$ and so $c = 3$ is a confluence of $\binom{3}{2} = 3$ simple crossings. While $e = 2$ and so $c = 4$ is a confluence of $\binom{4}{2} = 6$ simple crossings.

Remark 3 We target this by horizontally spreading out the vertical pairs of points. All the while preserving the top and bottom lines. With $\mathbb{Z}$-valued shifts so as to continue to reside within the square-grid presentations. In this manner, we attain Subfig d)'s general-position bipartite presentation on the 13×5 grid that we then use in our Introduction. We proceed by first spreading out the most highly nongeneric case.

Remark 4 The procedure used does not depend on the height. Whose somewhat larger value than the minimum 1 is chosen as part of carefully tracking the crossing points. The other part of this is the fineness of the lines drawn. We use these features to clearly display that the local cluster of crossing points above-left from the centre-bottom vertex are indeed in general position. Forming a fox-head-shaped cluster of 6 simple crossings. This pattern is also realized by the Pasch configuration mentioned in the preceding Article.

Open Questions 1-2 Is 13 the minimum grid-width? Does this minimum grid-width general-position bipartite presentation problem admit a unique solution?

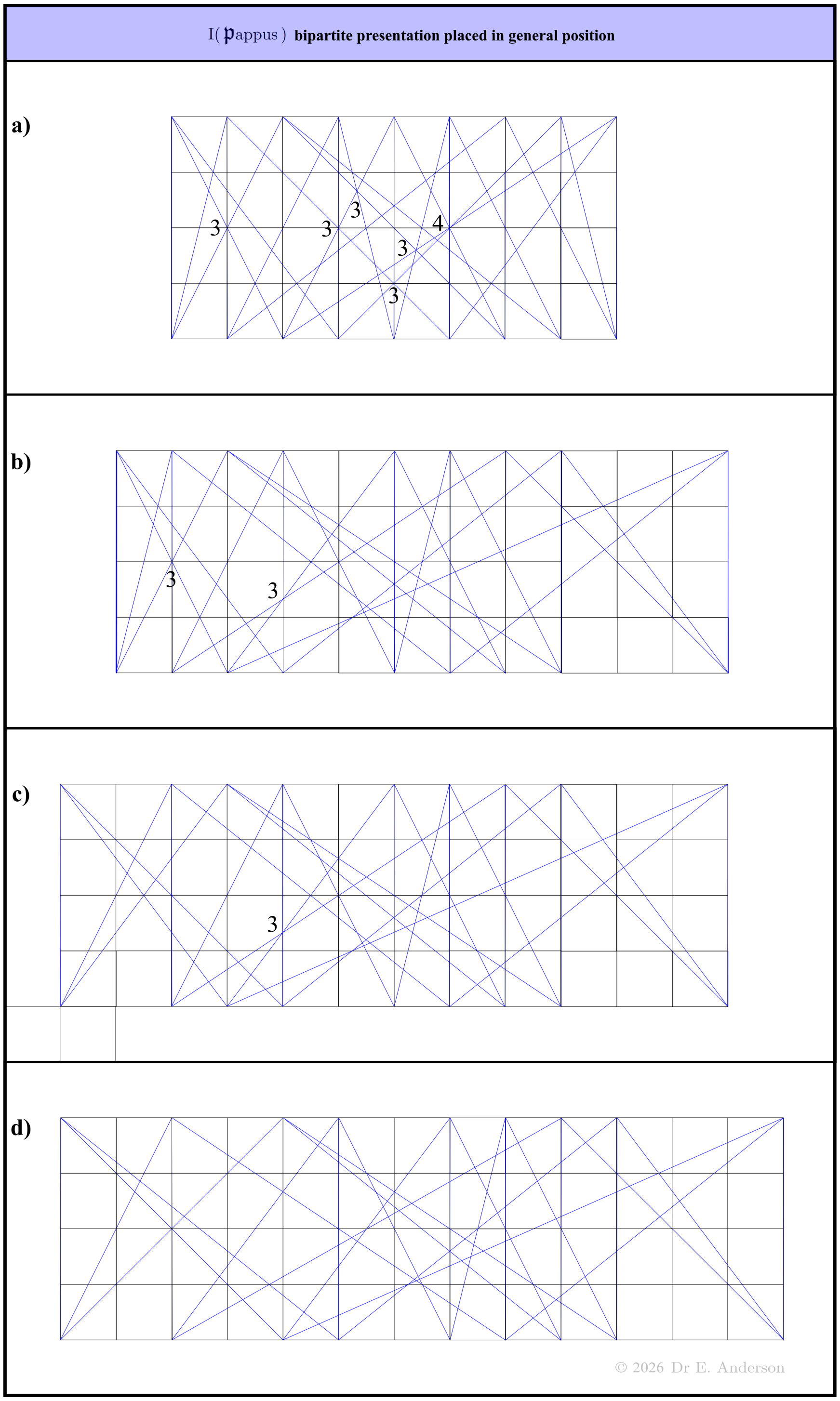
I(pappus) bipartite presentation placed in general position
a)
3
3
3
4
3
3
b)
3
3
c)
3
d)
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Figure 8:

A.2 Shellings

Remark 1 We exhibit here the square and hexagonal shells alluded to in the main text. Occasionally with the centrepoint marked in as an innermost zero-sized shell.

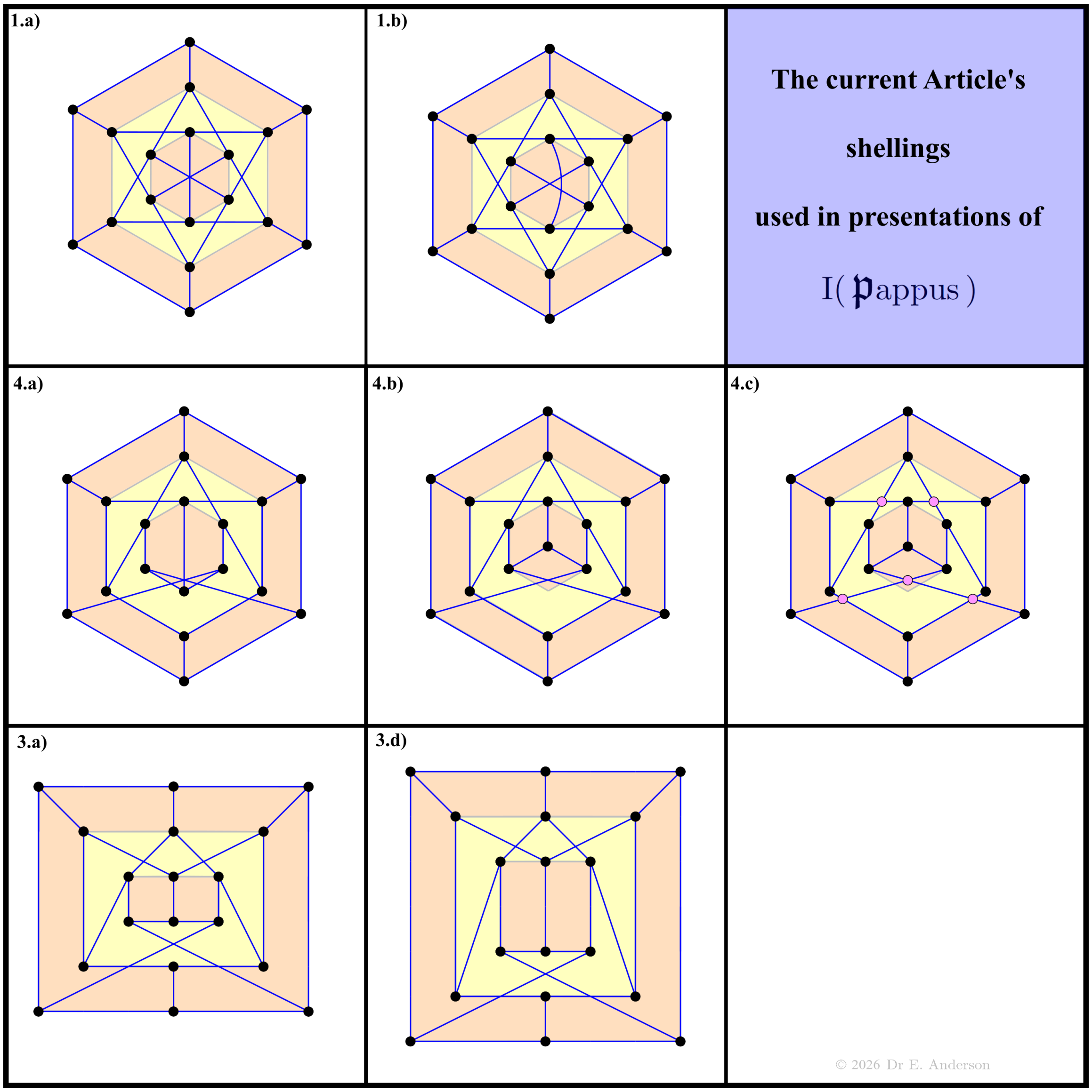


Figure 9:

A.3 Other tessellations

Remark 1 We finally exhibit here the equilateral-triangle and square tessellations alluded to in the main text. Observe that our mighty crab's edges themselves lie on a corresponding patch of right-isosceles triangle tessellation of the plane!

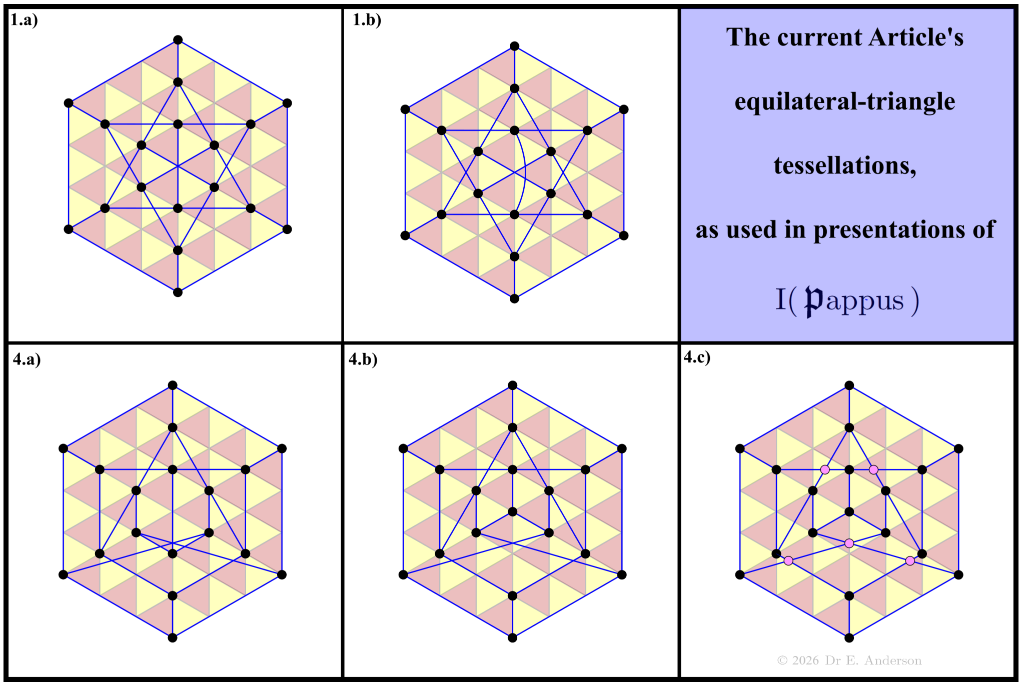
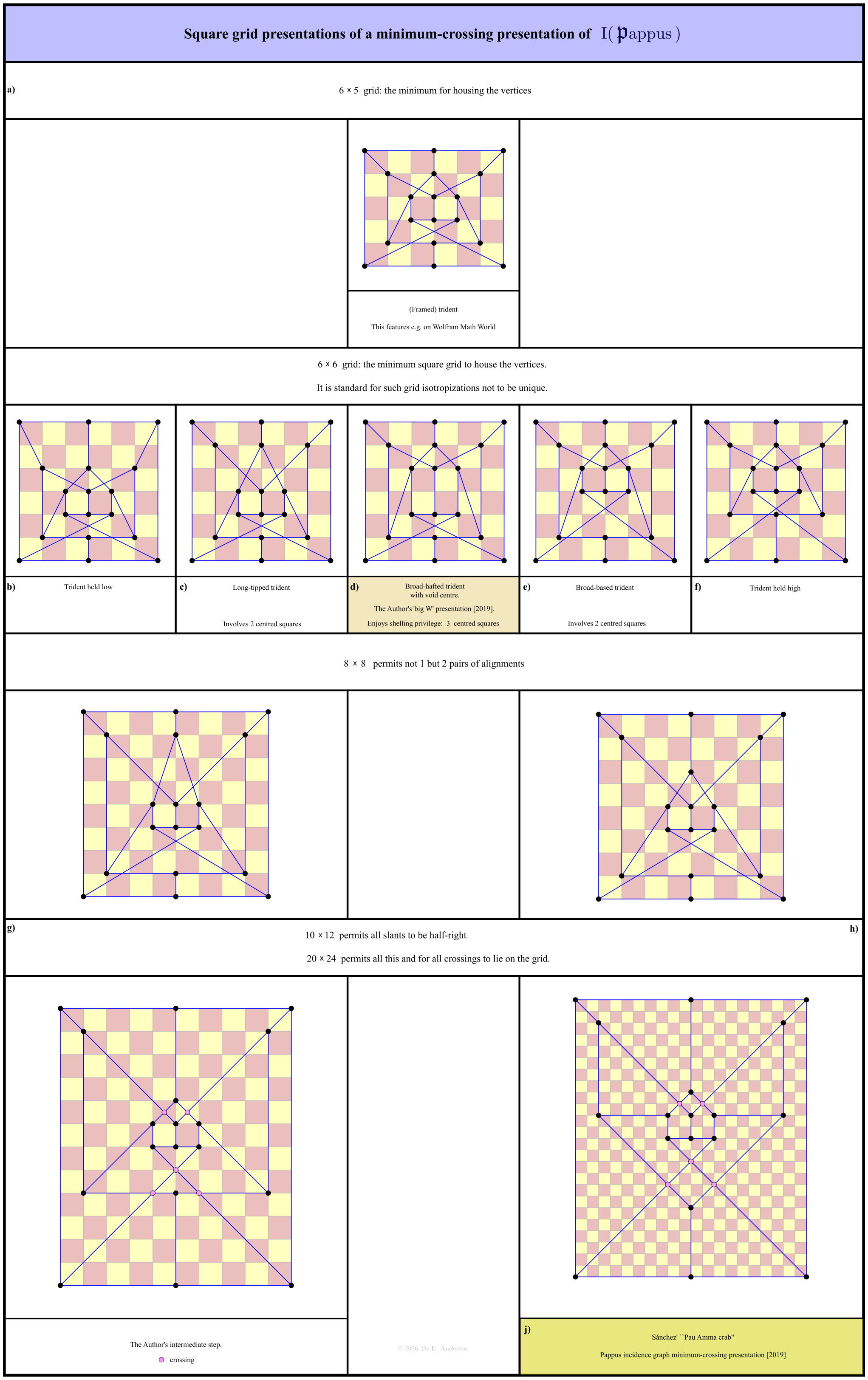


Figure 10:



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The Author's intermediate step.
crossing
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j)
Sánchez' ``Pau Amma crab"
Pappus incidence graph minimum-crossing presentation [2019]

Figure 11:

A.4 Call-signed version of our presentational medals table

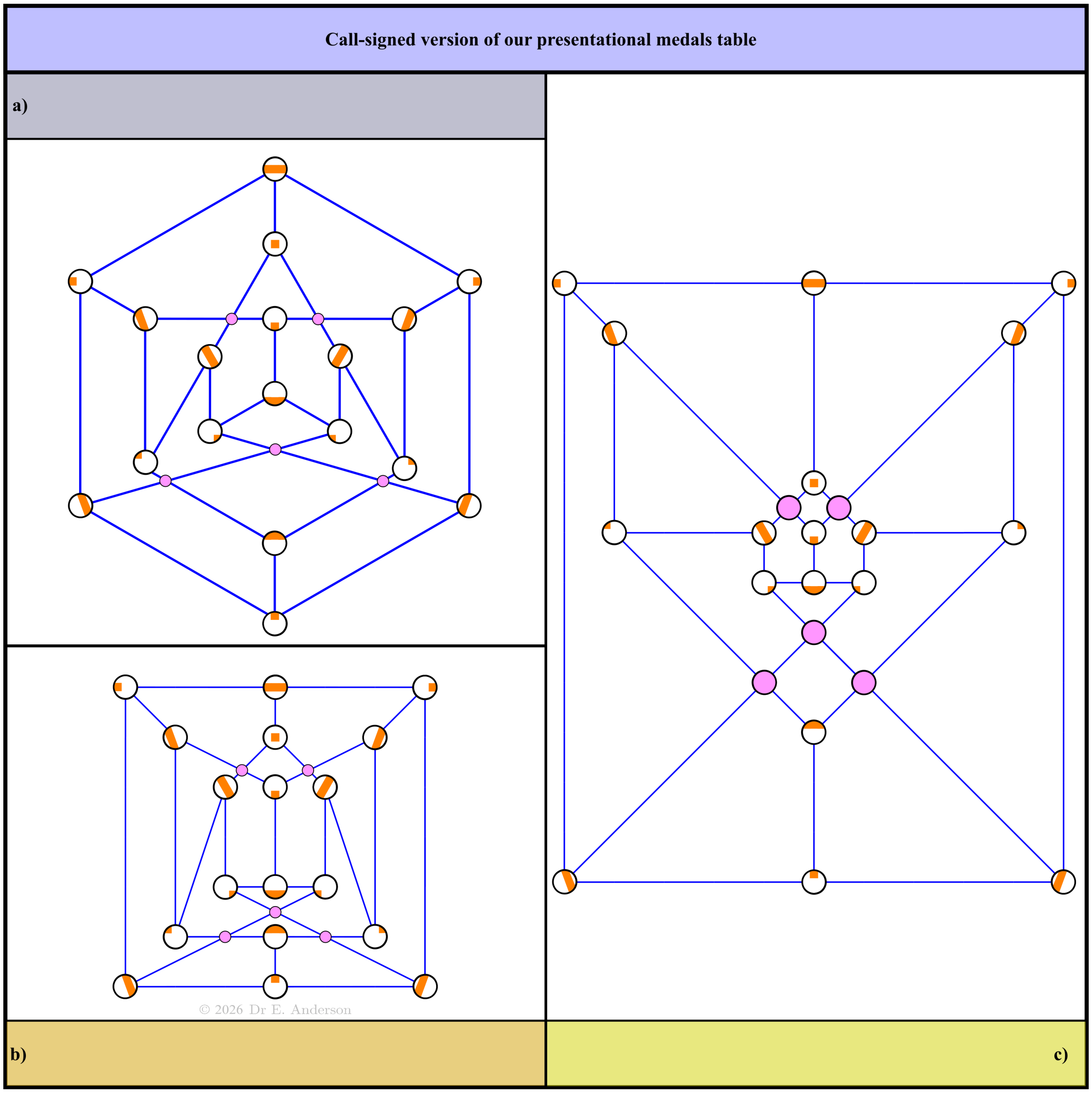

Figure 12:

Remark 1 This is in Fig 12. To celebrate c)'s nice placings of crossing points as well as of vertices, we here use the same size for the symbols for each.

Remark 2 We finally indicate where the yellow and green inputs, and the red output, reside in Fig 13

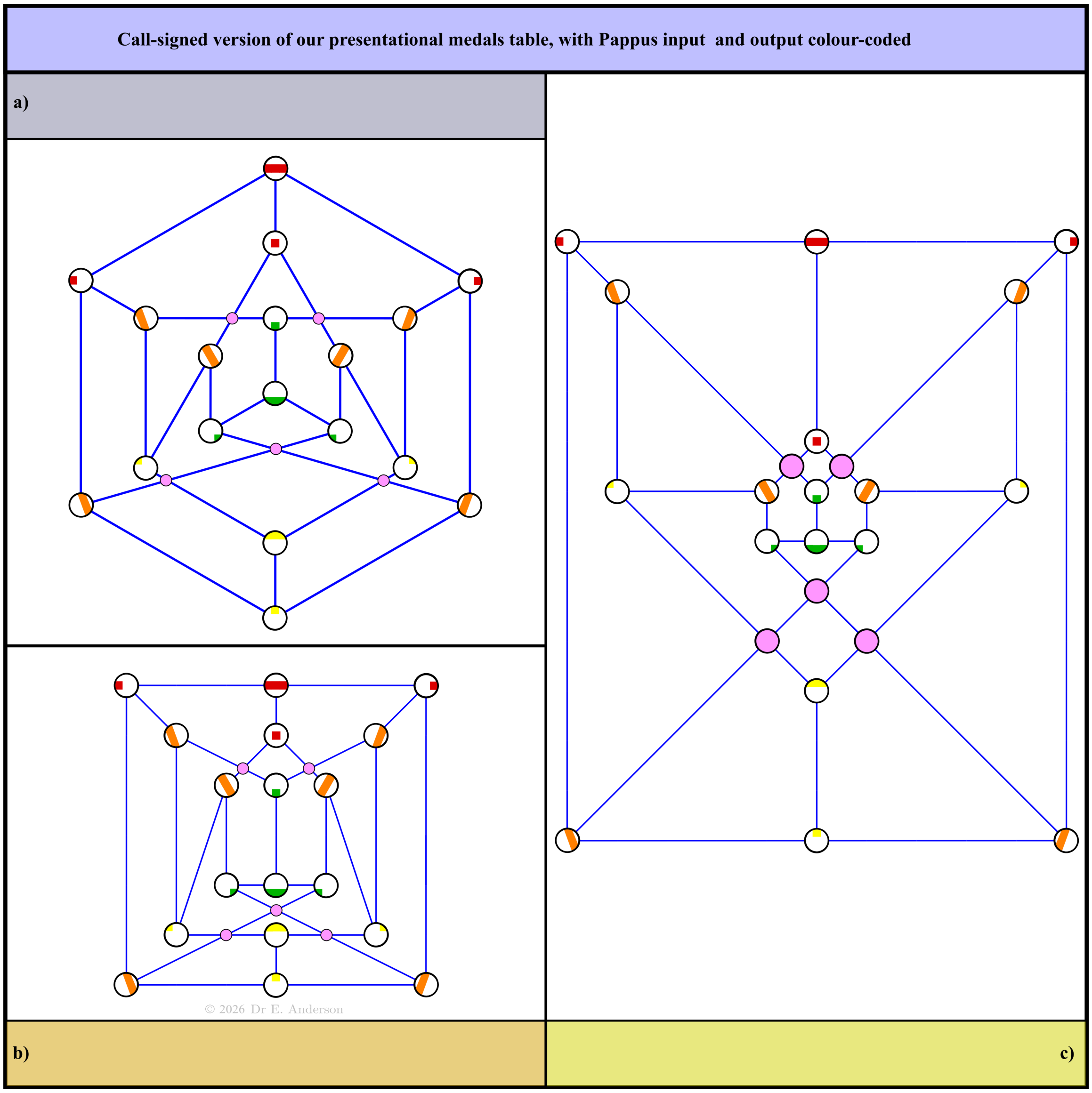
Call-signed version of our presentational medals table, with Pappus input and output colour-coded
a)
© 2026 Dr E. Anderson
b)
c)

Figure 13:

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