The Pappus Configuration, Theorem and Graph

Edward Anderson*

Abstract

We consider Projective Geometry's Pappus configuration, to which Pappus' Theorem applies, at the level of graphs. We select various conceptual classes of nice presentations for this graph. We finally point forward to various natural successors to this configuration, fundamental Projective theorem and graph.

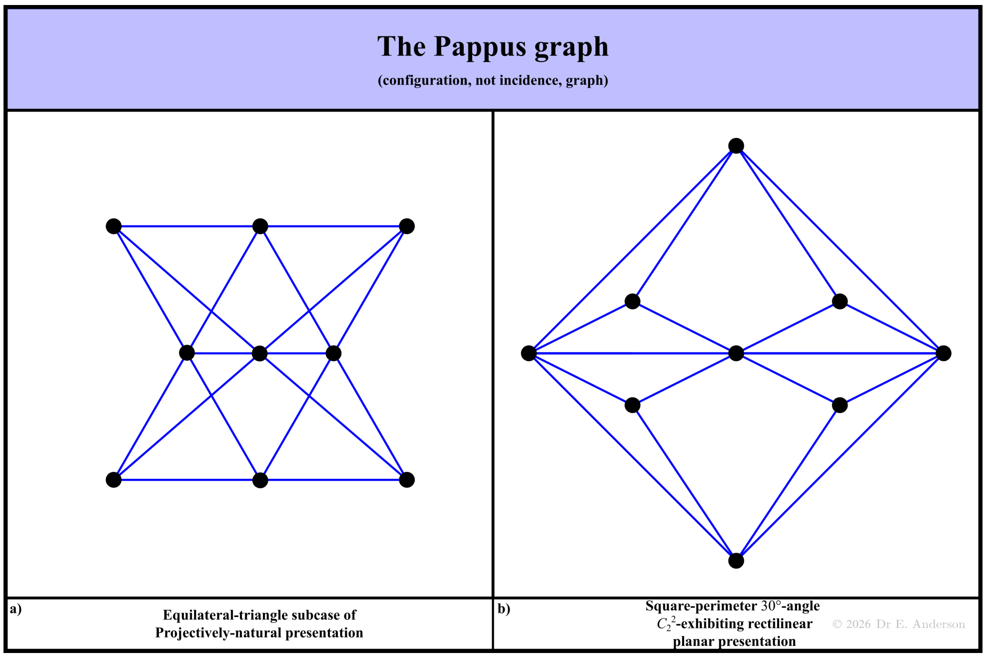

Figure 1:

This Article is ⟨3⟩: suitable for third-year undergraduates.

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* Dr.E.Anderson.Maths.Physics *at* protonmail.com . Institute for the Theory of STEM

1 Introducing Pappus' configuration and theorem

1.1 Pappus' Theorem in the Euclidean plane

Pappus' Theorem [2, 18, 17, 46, 42] Let $\mathrm{A}, \mathrm{B}, \mathrm{C}$ be a collinear triple of points in $\mathbb{R}^2$. And let $\mathrm{A}', \mathrm{B}', \mathrm{C}'$ be another. Then

$$\mathrm{AB}' \cap \mathrm{BA}' := \mathrm{I} \quad \text{and} \quad \text{3-cycles} \tag{1}$$

are themselves a collinear triple.

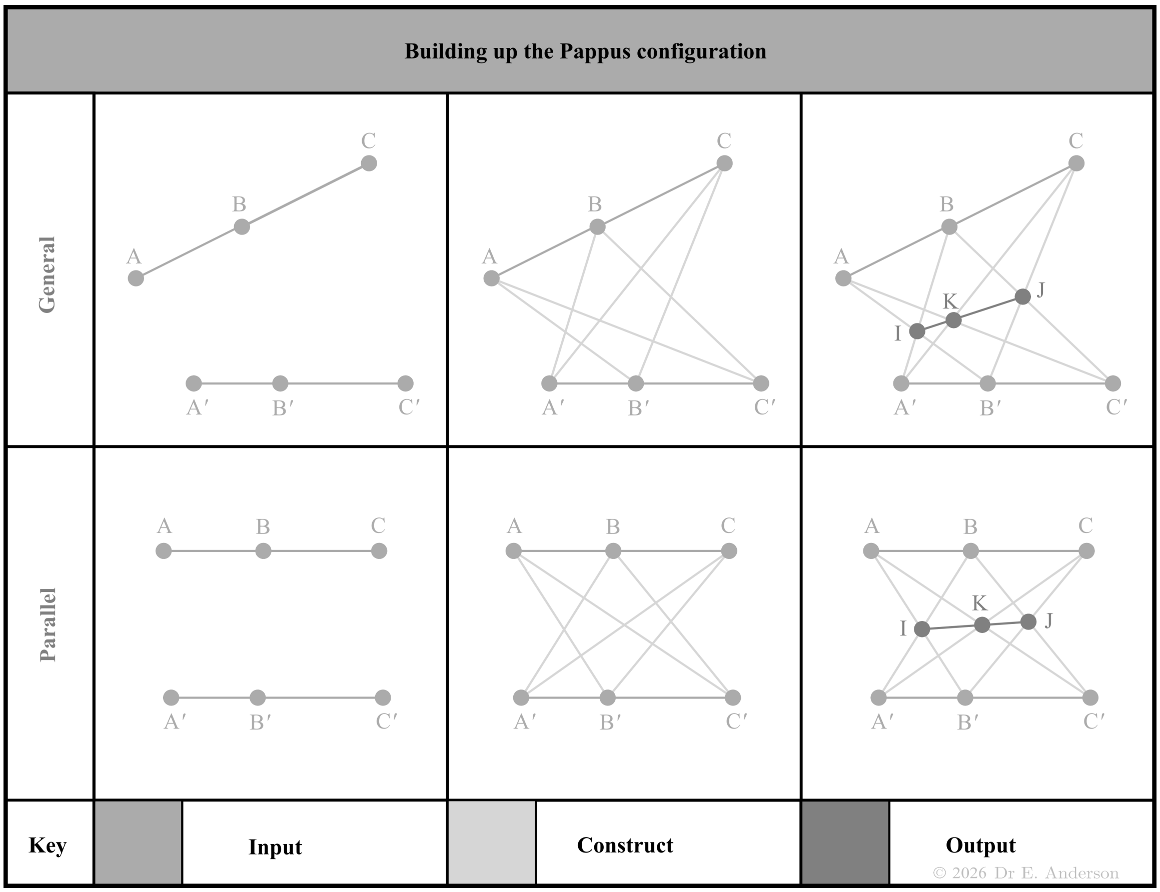


Figure 2:

Remark 1 See row 1 of Fig 2 for the general input construction and output. And row 2 for their counterpart when the 2 input triples are furthermore parallel.

Remark 2 One classical approach [22, 46, 42, 56] to proving this makes multiple uses of *Menelaus' Theorem.*[1]

1.2 Pappus' Theorem reappraised within Projective Geometry

Remark 2 This late Ancient Greek result harbours substantial Projective-Geometric significance. Though recognizing this had to await the 19th [7, 8, 9] and even 20th centuries [10, 27, 39]. In particular, it turns out to constitute one of the two main structural theorems of Projective Geometry in flat space. *Projective Geometry* [39, 20, 21, 11, 17, 25] is the study of *incidence*. At the level

[1] See whichever of [19, 13, 12, 36] for an introduction and [22, 24, 30, 56] for discussions and applications.

of planes, this is a binary relation on points-and-lines. Comprising whether points lie on lines and whether lines intersect each other.

Structure 1 An abstract Projective plane [51] is *Pappian* if Pappus' theorem holds universally throughout it. Elsewise it is *non-Pappian*.

1.3 Pointers to other styles of proof

Pointer 1 ⟨0-3⟩[2] By multiple uses of [42] the basic *side-split lemma*. Alias *side-splitter*, *intercept* or *basic-proportionality lemma* or *theorem*, or one of the *theorems of Thales*. Or by an area-sum method [42]. Alongside Menelaus, these were all 'classically available' methods. A further pre-Projective approach [42] is to make multiple uses of Hud–Ceva's theorem in place of Menelaus'. In various senses Menelaus and Hud–Ceva are dual statements [56].

Pointer 2 It is straightforward to obtain a Projective proof intrinsically within[3] 2-d ⟨3⟩; see for instance [21, 17]. Or by using cross-ratios [44]. Or by composing projective transformations [35].

Pointer 3 Algebraic approaches include use of homogeneous coordinates [42]: one of the most common types of Projectively-significant coordinates ⟨3⟩. This particular reference is further laced with Linear Algebra by its use of determinants. Though one can proceed instead using [42] ⟨3⟩ *Plücker relations*: a Projective notion. Brute-force Vector Algebra will also do [42]. These things said, proving Pappus' Theorem need not be a lengthy or technical affair, see e.g. [38] for a simple Coordinate Geometry proof ⟨2⟩.

Pointer 4 ⟨3-4⟩. Once various more substantial Projective theorems are available, Pappus' theorem drops out as a corollary. For instance from *Pascal's theorem*, by specializing its conic to a pair of intersecting lines [28]. Or from Desargues' theorem [39], by specializing its pair of triads to be in perspective.

1.4 Pedagogical aside

Pedagogical Remark 1 Observe however that just *2 to 4* styles of proof for Pappus' theorem suffice up to ⟨3⟩ . Say a classical proof in ⟨2⟩, though in particular the most pedestrian – side-spliter – could be attempted in some earlier year. And a Projective proof in ⟨3⟩, alongside a Linear Algebra proof to illustrate the ongoing usefulness of the Linear Pillar of Geometry. Those following Stillwell's [39] Four Pillars of Geometry might pointedly want to extend your repertoire to a fourth Transformation Groups proof...

Pedagogical Remark 2 More proofs than this would be of particular interest when the Reader is doing 1) a graduate-level course in Projective Geometry. 2) A Ph.D. specializing in Projective Geometry. 3) Or a thesis or research project specifically on Pappus' theorem, or on some small set of Projective theorems that include Pappus'. Such as the Pappus–Desargues fundamental pair, or the Pascal–Brianchon dual pair [14, 22], which connects to Pappus' theorem as per below. With some tongue in cheek, passing from Stillwell's 4 Pillars of undergraduate Flat Geometry to a larger number of Pillars beyond, includes the following. Geometrical invariants themselves come to constitute a Pillar, with various further Pillars [61, 62] providing first principles either for obtaining these or for obtaining the transformation group. As such, the graduate with reasons to study Pappus' theorem further might do well to next look into a cross-ratios proof...

[2] The current Encyclopedia [49] uses chevron brackets to denote "approximate year of study".
[3] A larger proportion of sources prove the other main theorem of Projective Geometry – Desargues' – by exiting from 2-d into 3-d .

2 Introducing the Pappus graph

2.1 The Projectively-natural presentation

Remark 1 Take a particularly symmetric version of the affinely-special subcase of Pappus configuration. This is Topologically equivalent to all the others, while making for nice presentations [45]. One of these uses the equilateral triangle grid. I.e. a patch of the tessellation [29] of the plane by equal-sized equilateral-triangle tiles. See Fig 1.a). While Coxeter [16] uses a symmetrical Affine presentation of Pappus, superposing our figure reveals that his triangles are slightly taller than our equilateral ones...

Aside 1 This is not to be confused with the *Pappus incidence graph*, to which we return in the Conclusion's pointers...

2.2 Some basic counts and properties

Remark 2 So the Pappus graph has

$$V(\text{Pappus}) = N = 9 , \tag{2}$$

$$E(\text{Pappus}) = 18 . \tag{3}$$

This corresponds to an average of exactly 4 edges per vertex.

Remark 3 The Pappus graph's degree sequence is

$$\deg(\text{Pappus}) = 3^4\, 4^2\, 5^2\, 6 . \tag{4}$$

The Pappus graph is consequently neither *regular* [48]: of a single degree. Nor a *cone* [48]: with ≥ 1 degree-$(V - 1) = 8$ vertices. It does however have a sole vertex with greater degree strength than the others: 6. Which we subsequently prefer to place centrally in forming nice presentation.

Remark 4 It is straightforward to show that the Pappus graph is *planar* [26, 37]. Given Fig 1.a)'s presentation 'in the wild', 4 identical tucks of the degree-3 vertices will do. The square-perimeter 30-degree angle presentation of this is exhibited in Subfig b).

Remark 5 The Pappus graph then clearly has

$$F(\text{Pappus}) = 10$$

internal faces Or a total of

$$F_{\text{T}}(\text{Pappus}) = 11$$

faces including the external face.
In the first interpretation,

$$V - E + F = 9 - 18 + 10 = 1 = \chi(\mathbb{D}^2) :$$

the Euler characteristic of the disc. In the second interpretation,

$$V - E + F_{\text{T}} = 9 - 18 + 11 = 2 = \chi(\mathbb{S}^2) .$$

Where $\mathbb{D}^2$ is the disc and $\mathbb{S}^2$ is the sphere.

2.3 Grid and tessellation presentations

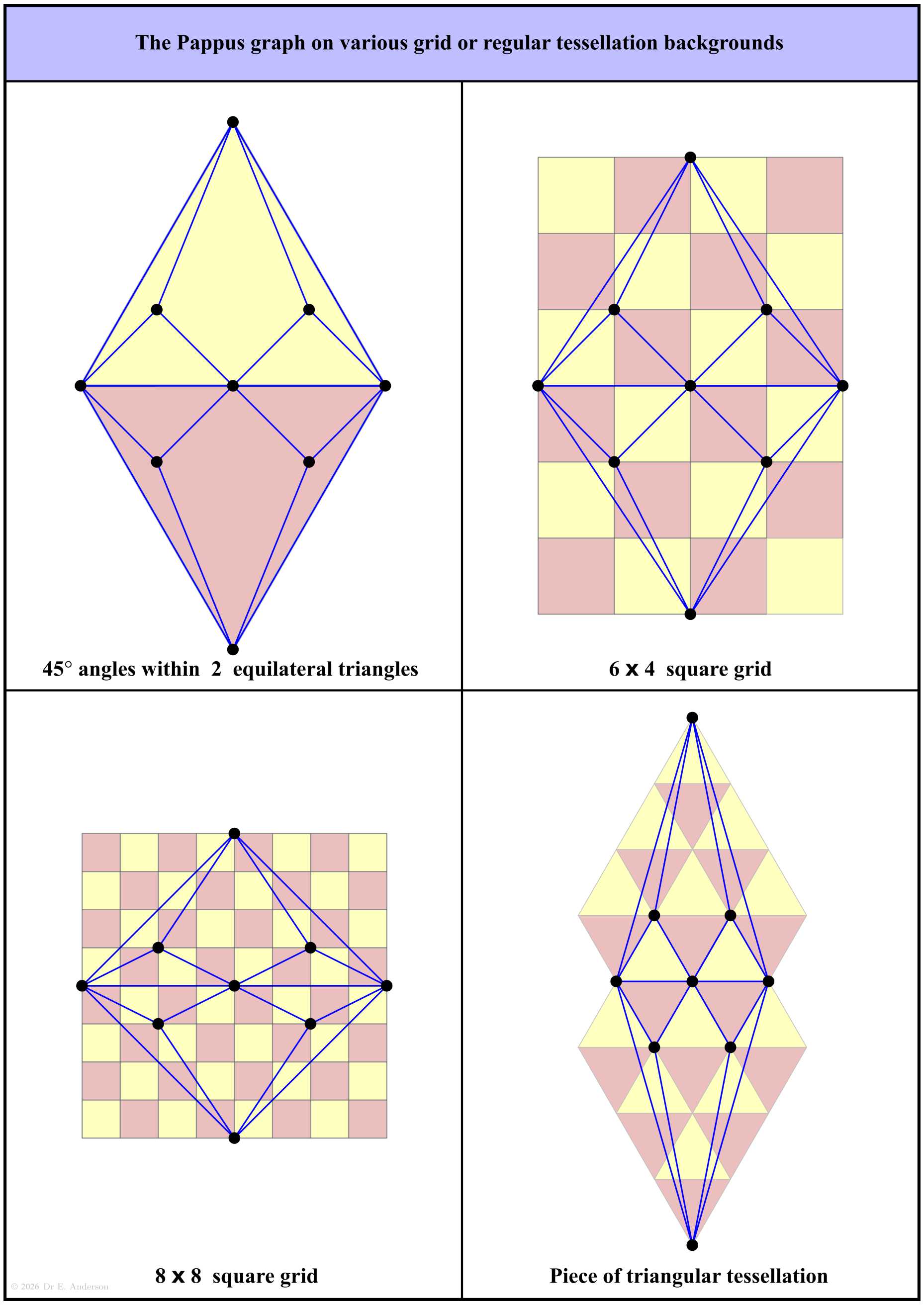


Figure 3:

Remark 6 We provide 4 of these in Fig 3.

2.4 Ramsey presentations

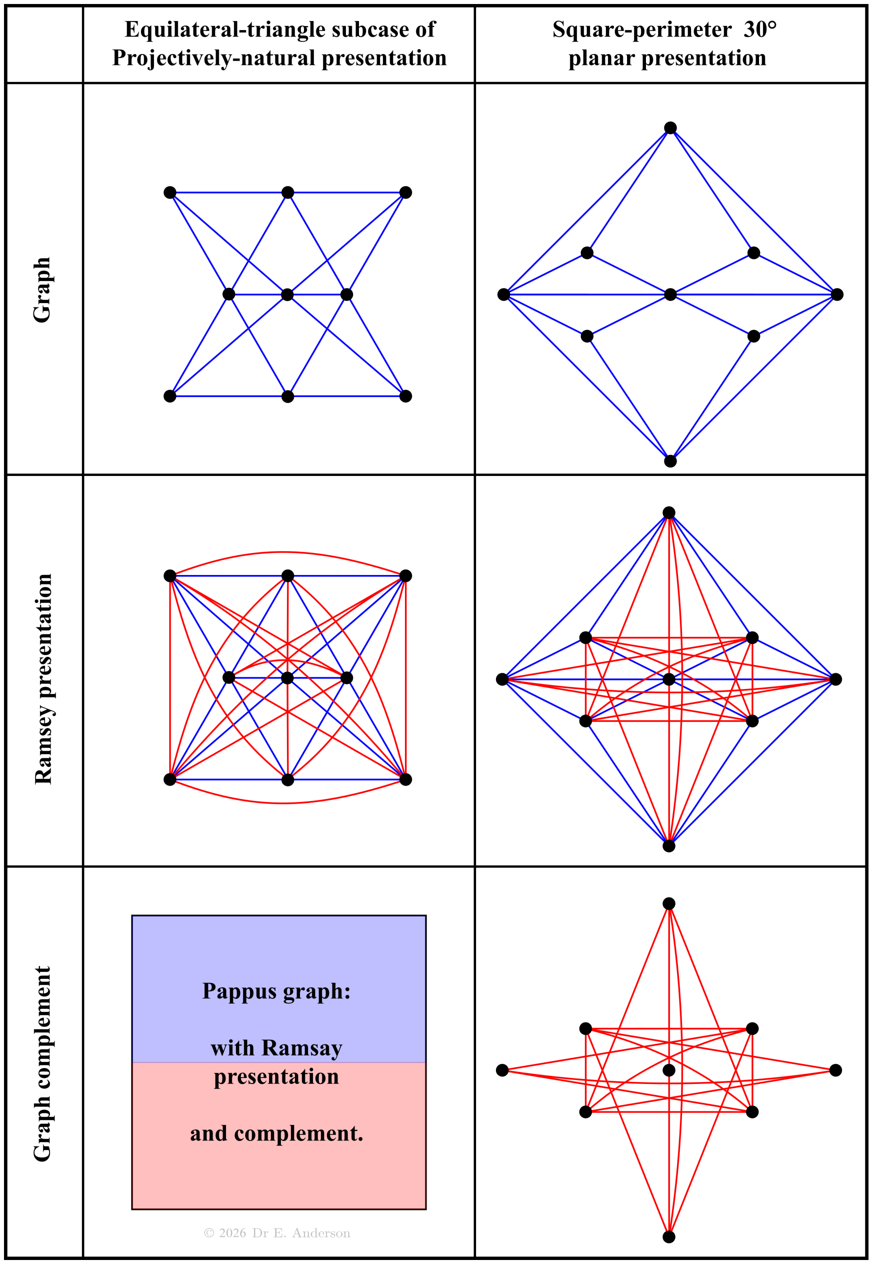


Figure 4:

Structure 1 See the second row of Fig 4 for the corresponding *Ramsey presentations* on each of the above 2 presentations' blue chassis. This builds in a simple graph's non-edges in red: on an equal footing with the edges in blue.

Remark 7 One can then peel off the blue to leave the red to subsequently unfold, revealing the structure of one's incipient graph's complement. See row 3 for the peeled version, and Fig 5 for unfolded versions.

2.5 The Pappus graph is edge-critical and heterocomplementary

Structure 2 A simple graph G is *edge-critical* if

$$E(\,\mathrm{G};\,N\,) \;=\; E_{\mathrm{crit}}(N) \;=\; \frac{V\,(\,V\,-\,1\,)}{4}\;. \tag{5}$$

This corresponds to a graph G and its complement $\overline{\mathrm{G}}$ having the same number of edges. This can only happen if

$$V \;=\; 0,\,1 \;\;(\,\mathrm{mod}\;4\,)\;. \tag{6}$$

Structure 3 A graph is *homocomplementary* if it is isomorphic to its own complement. Otherwise it is heterocomplementary.[4]

Diagnostic 1 The above simple graph edge-criticality conditions (5, 6) are necessary but not sufficient for a graph to be homocomplementary.

Remark 8 In fact, generically, edge-critical simple graphs are not self-complementary.

Diagnostic 2 A more detailed necessary but not sufficient condition for homocomplementarity is that the graph and its complement share degree sequence.

Remark 9 From (2, 3), the Pappus graph turns out to be edge-critical:

$$E_{\mathrm{crit}}(9) \;=\; \frac{V\,(\,V\,-\,1\,)}{4} \;=\; \frac{9\,(\,9\,-\,1\,)}{4} \;=\; \frac{9\,\times\,8}{4} \;=\; 18 \;=\; E(\,\mathrm{Pappus};\,9\,)\;.$$

And yet to be generic in the heterocomplementary sense rather than homocomplementary. For

$$\deg\left(\overline{\mathrm{Pappus}}\right) \;=\; 2\,3^2\,4^2\,5^4\;. \tag{7}$$

And this does not match (4). So our noted high-degree vertex having no balancing low-degree vertex suffices to dash any hope of the Pappus graph being in the more distinguished class of the heterocomplementary graphs.

Principle 1 Suppose that

$$E(\,\mathrm{G};\,N\,) \;>\; E_{\mathrm{crit}}(N)\;. \tag{8}$$

Then it is usually easier to understand the structure of $\overline{\mathrm{G}}$ than that of G . It is usually also more straightforward to name graphs via whichever of themself or their complement has smaller size (= edge number E) . For edge-critical graphs, there is no a priori reason for one to be more suitable than the other. See [55] for some further selection principles in this critical edge case.

[4] Hitherto, the alias *self-complementary* has been more widely used than homocomplementary.

2.6 A few properties of the Pappus complement

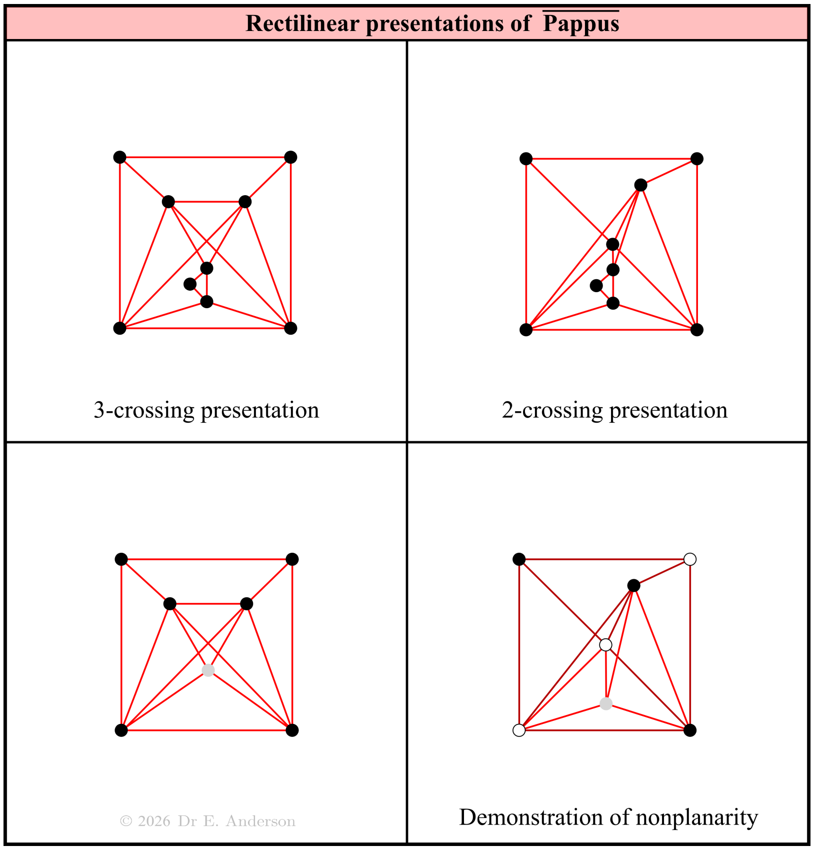


Figure 5:

Remark 10 We unfold row 3 of Fig 4's presentation of $\overline{\text{Pappus}}$ to quickly obtain a 3-crossing presentation in Fig 5.a). And then get it down to 2 crossings in Subfig b).

Subfig d) suffices to demonstrate nonplanarity: it contains, as a subgraph, the utilities graph Utilities $= K_{3,3}$ [48]. Where the second symbol denotes complete-bipartite with 2 pieces of order 3 . By Kuratowski's theorem [37], this is indeed one of the 2 irreducible forbidden subgraphs in a planar graph. The other is the complete graph K_5 .

3 The Pappus graph is a double irreducible

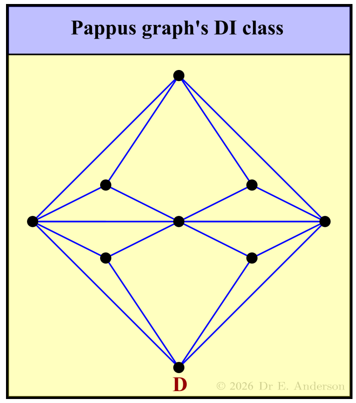


Figure 6:

Remark 1 The Pappus graph has no leaves. It is furthermore a homeomorph irreducible. So it is a double irreducible: class D (Fig 6).

4 Notions of traversability

4.1 The basic notions

Remark 1 The Pappus graph is immediately not *Eulerian*, since it contains odd-degree vertices.

Remark 2 The Pappus graph is straightforwardly *Hamiltonian*.

Remark 3 Consult Chapters 41 and 42 of the freely available [48] if you are not sure what these two notions of traversability mean, or what their basic conceptual content is.

4.2 ZIPHoN treatment

Remark 4 Since the Pappus graph is also planar, one can investigate its Hamiltonian properties using the *ZIPHoN theorem*[5] By this result, every Hamiltonian cycle splits the graph into 2 outerplanar strips. Each containing an equal amount T of triangulating triangles.

Notational Remark 1 We colour in one outerplanar strip in yellow, leaving the other in white. This includes modelling yellow and white as individually-meaningless and yet mutually-distinguishable labels. By which such strips are invariant under colour exchange. Due to this, one can w.l.o.g. never colour the outer face. We finally mark the bounding Hamiltonian cycles using thick emerald edges.

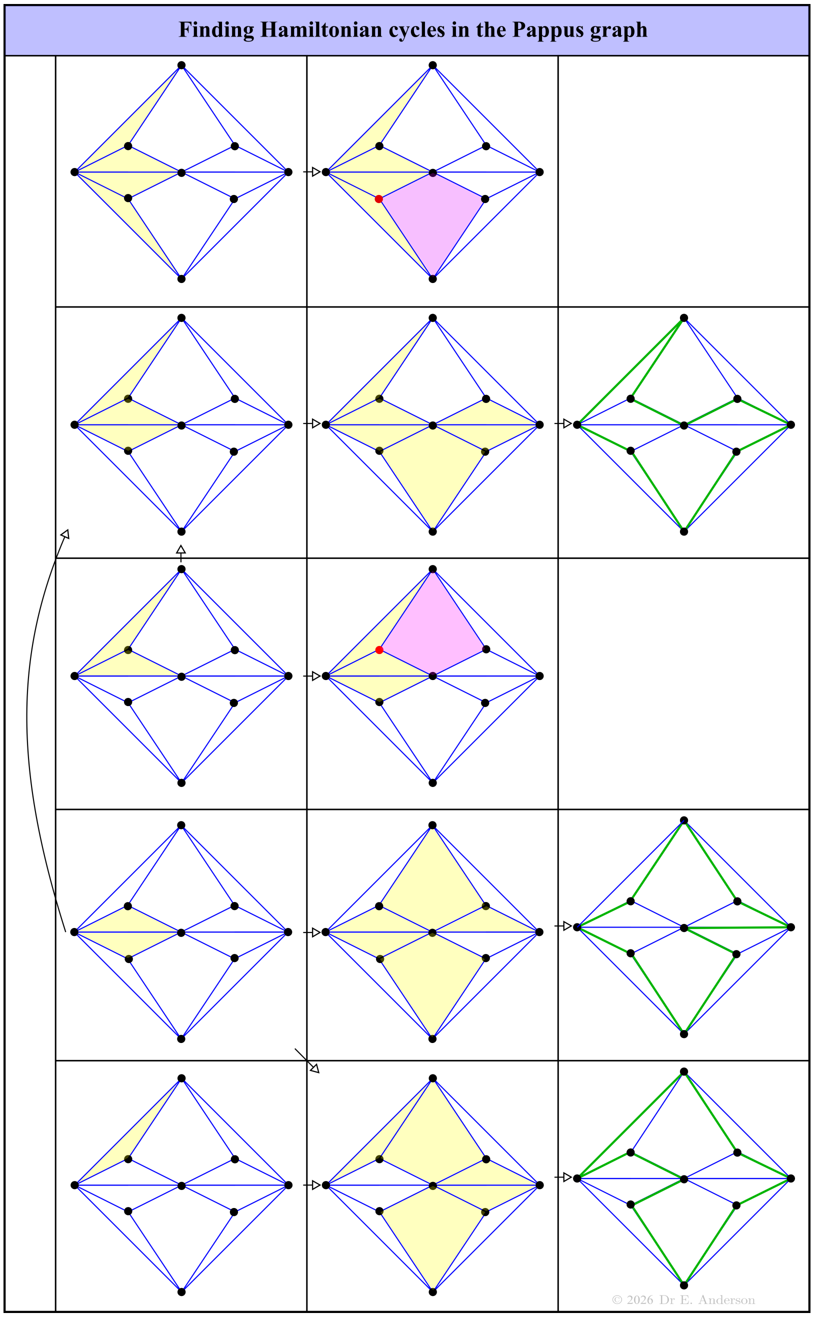
Remark 5 For the Pappus graph,

$$T = 7 .$$

Fig 7 produces 3 such strips, while also indicating some of the forcings by which no more cases are possible.

Remark 6 Our exhaustive case location procedure is to start with a fan of 4 triangles. This fails to give any cases as per row 1. We next consider a continuous fan of 3 triangles, giving the second rows' case. We next consider a contiguous pair, one with a degree-6 vertex and the other not. This is forced either to return to the previous or to give the third row's case. A contiguous pair with both sharing the degree-6 vertex gives our last case. All other cases – 1 or 0 triangles within such a fan – just either become impossible. Or return previous cases, under colour reversal if needs be.

[5] Zero-index planar Hamiltonian necessity theorem [47, 48, 53]. Alias Grinberg's theorem [23, 31, 37].



Finding Hamiltonian cycles in the Pappus graph
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Figure 7:

4.3 Manifestly Hamiltonian presentations

Remark 7 Corresponding manifestly-Hamiltonian presentations of the Pappus graph are given in Fig 8.

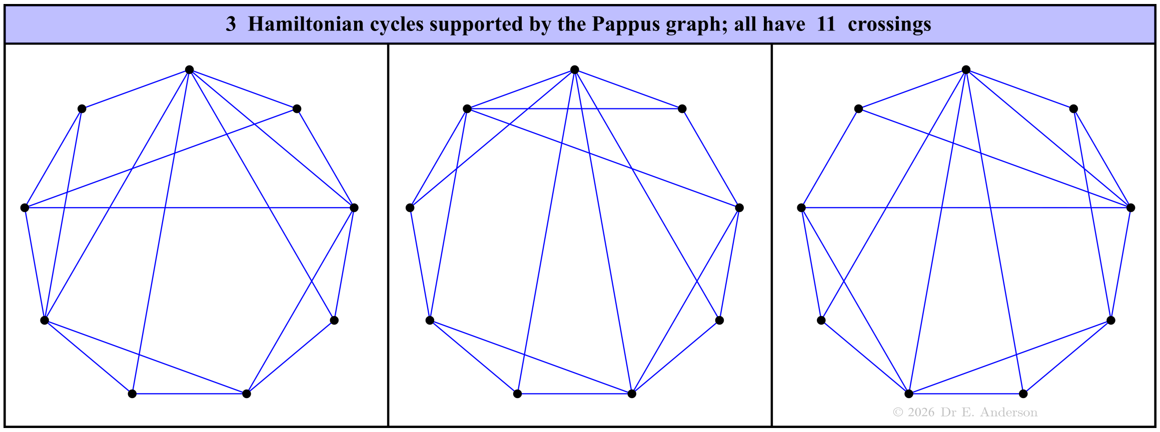

Figure 8:

Remark 8 The idea here is to use a manifest regular N-polygon perimeter in place of picking out a non-obviously realized N-cycle in emerald.

Remark 9 We finally make use of both the inner face and the outer face to retain manifest planarity in Fig 9.

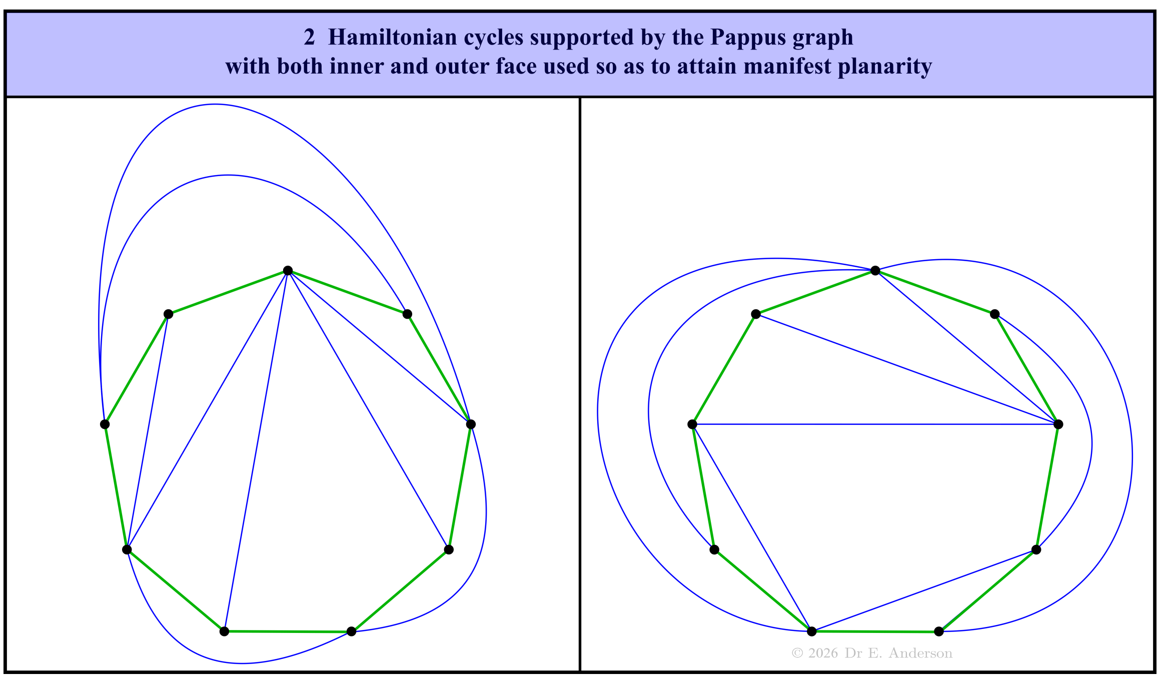

Figure 9:

5 Notions of colourability

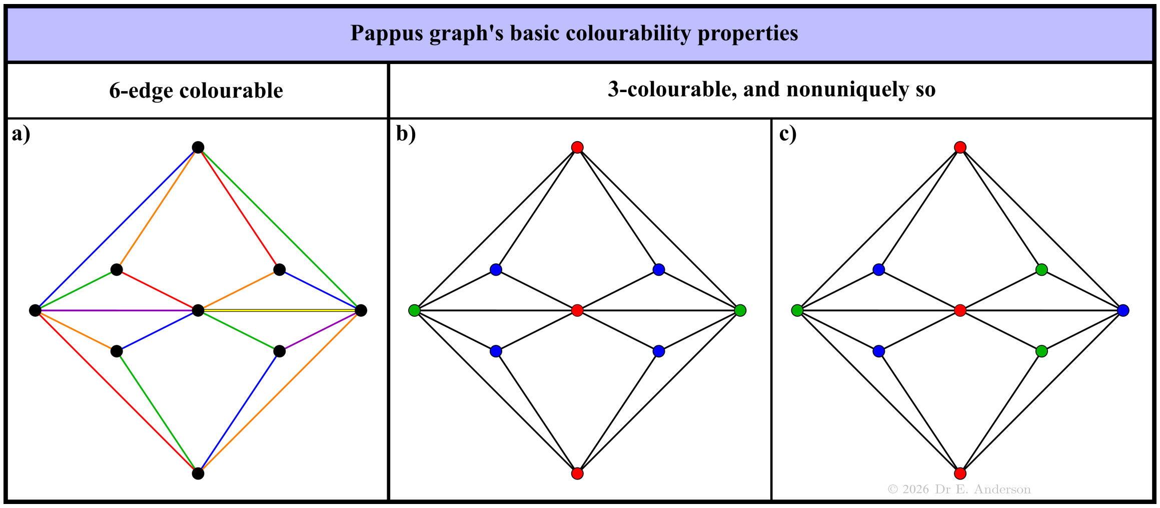

Figure 10:

Remark 1 Since the Pappus graph has a degree-6 vertex, it is $\geq$ 6-edge colourable. In fact, it is precisely 6-edge colourable, as per Fig 10.a).

Remark 2 Since the Pappus graph contains triangles, it is $\geq$ 3-colourable. And since it is planar, it is $\leq$ 4-colourable by the famous 4-colour theorem. In fact, it is 3-colourable, as per Subfig b).

Remark 3 See e.g. [26, 37] or Chapter 47 of the freely-available [48] for an introduction to these notions of colorability. The first two of these are also useful first references for the 4-colour theorem. See [32] for a more advanced account of colourability.

Remark 4 With the Graph Atlas [34] tabulating unique colourability as well, we draw out Subfig c) to demonstrate that the Pappus graph does not have this further property.

6 Pointers to allied material

Pointer 5 A more systematic study would start with Pappus' law's simpler configuration. This shall be covered within our article on the simplest Projective graphs [57]. Where no Projective plane conditions, or adjectives, are required. This shall for instance also cover scissor [39] graphs.

Pointer 6 A subsequent systematic study 5 should also cover degenerate cases, such as Pappus' little theorem. And other variants, such as including the intersection point(s).

Pointer 7 Incidence is the central notion in Projective Geometry. The *incidence graph* for the Pappus configuration shall first be covered in 4; Graph Theorists often call incidence graphs *Levi graphs*. They and Geometers often call the Pappus incidence graph the Pappus graph. Whereas the current Article takes this name to mean the Pappus *configuration* graph.

Pointer 8 The Pappus configuration is double-tied in third place [33] for (one notion of) Projective configuration. The smallest is the Fano plane: $N = 7$ [51]. Next there is a unique projective plane for $N = 8$. And then Pappus arrives, accompanied by 2 other 9's. [60] shall more eventually cover the 8 and the 2 other 9's as graphs; for now see [33, 41, 43] at the level of configurations.

Pointer 9 The *Desargues configuration* is then 1 of the 10 10's. The corresponding *Desargues' theorem* [5, 15, 21, 18, 25, 28, 38, 35] is *the other* fundamental theorem of Projective Geometry. The Desargues configuration, theorem and graph are covered in [54]. Again, this is not to be confused with a more famous incidence graph, which is covered instead in [58]. Non-Desarguian Projective planes [40] are more often covered than non-Pappian ones.

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