2024 SUMMER SCHOOLS

at The Institute for the Theory of STEM

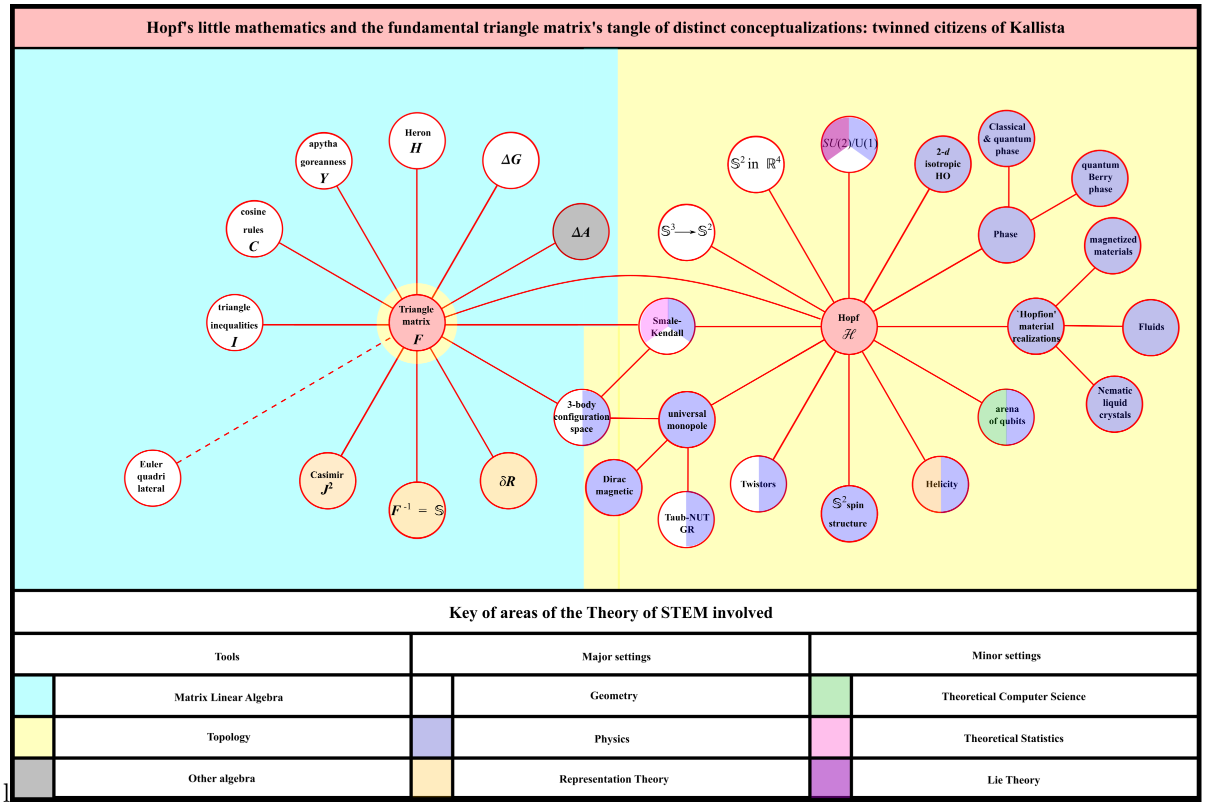

Figure 1:

New results in the geometry of triangles and quadrilaterals (July 2024)

This is the continuation of the Applied Topology and Combinatorics discussion group's topic so far in 2024.

1) Linear Algebra of the 4-body problem.

2) How Apollonius' Theorem and Euler's Quadrilateral Theorem (EQT) generalize to all subsequent eigenclusters.

The stated two being the 3-path and 3-star cases.

3) The arena of triples of Cevians.

4) Linear Algebra of Ptolemy's Theorem (including in combination with EQT).

5) Linear Algebra of Brahmagupta's, and Bretschneider's second, quadrilateral area formulae.

The latter is alias Coolidge's area formula, but at the Linear Algebra level, it is identical to Bretschneider 2 !

6) Linear Algebra of further quadrilateral area formulae.

Including why this is often addressing the wrong question. For Heron's formula is tied to a Casimir, and the quadrilateral analogue of this is *not* an area formula for the quadrilateral. It is, rather, the sum of the squares of a given clustering's 3-subsystems! By which, at some point in 2026, there will be an extra Seminar 7), completing this material!

The Hopf Map 30+ times in Physics and Geometry (August–September 2024)

The fundamental triangle matrix $\boldsymbol{F}$ has already been covered in the Applied Topology and Combinatorics discussion group. The Heron–Hopf–Smale–Kendall bridge connects this many-routes object with the even-more-routes-structure of Hopf's Little Mathematics. Which Penrose characterized as ""element of the architecture of our world". Sánchez called both $\boldsymbol{F}$ and the Hopf map citizens of Kallista. Which is not exactly the same thing, as shall be gradually discussed...

1) The fundamental triangle matrix: truer names as tempered by yet further occurrences.

Concerning its also being the Newton–Euler–Gauss–Jacobi-H matrix (ouch). While not in general being a spherical- or hyperbolic-triangle structure. So one should say fundamental *flat*-triangle matrix. Including also how the triangle inequality – cosine rule – Heron's formula coincidence for flat triangles breaks down for spherical and hyperbolic triangles. And yet $\boldsymbol{F}$ recurs in the leading-order approximation to the spherical and hyperbolic cases of the cosine rule. With the spherical such admitting a beautiful projector formulation!

2) The Heron–Hopf–Smale–Kendall bridge.

3) Hopf's original working and Kendall's original working do not overlap.

2) and 3) double as our next test subject for Testarossa: the colour-coded notation so as to not confuse index types nor which spaces objects belong to.

4) Fibration and fibre bundle interpretations of the Hopf map.

5) Clifford tori and Penrose's depiction of the Hopf map.

6) The Dirac monopole

7) ... and its many 3-body problem friends.

Most distinguished among which are the Iwai monopole, its 2-d analogue and the 'Leibniz–Kendall' and 'Leibniz–Whitney–Kendall' monopoles.

8) The Taub–NUT solution of GR is underscored by the Hopf map.

9) The space of qubits.

10) Hopfions.

These being the other main development in applications of the Hopf map in the past decade, aside from our monopoles and fundamental triangle matrix work!

11) Discussion on Urbantke's account of a few of the more technically-involved applications of the Hopf map.

Meaning helicity, spin structure and Penrose's twistors!